The Abandonment Option Value of Public Private Partnership Projects in Brazil

Summary
Pricing techniques are crucial to value embedded flexibilities in Public Private Partnership (PPP) projects. Even when PPP contracts are clearly understood by the parties, uncertainties are commonly not well measured. Recently, unsuccessful PPP bids in Brazil indicated the increase of the uncertainty perception over capital investment discourages bid attendees and also increase the project abandonment probability. In that context, we developed a real option model to value the flexibility of sequential investments with imbedded option to abandon the project. The capital expenditure of each phase of the project follows a Geometric Brownian Motion process. We apply the model to a bidding process of broadband infrastructure project in Brazil. We calculated the value of the real option as US$9.09 million, which represents up to a 31% increase over the project baseline value obtained by the DCF analysis. We simulated the probability of abandonment as a function of the penalty fee and CAPEX volatility. The project risk perception were simulated considering the probability of project abandonment, which achieved 12.5% in the first year and it decreases substantially in the next four years of the construction phase; it is less than 0.5% in the 2nd year and less than 0.7% in the 3rd year. The probability to abandon the project at years 4 and 5 are virtually zero. This model can allows governmental policy-makers to design PPP projects with the most appropriate penalty fee schedule and provides a quantitative tool to disincentive the abandonment of the concessionaire in PPP projects.

Key words: Contract flexibilities, Uncertainty, Public-Private Partnership, Real options.

1 Introduction
Public infrastructure projects are impacted by large initial capital investments, the presence of idle capacity and the demand has low sensitivity of demand to changes in prices. Nevertheless, operational margins tend to be higher and revenues tends to be more predictable than other industrial sectors (Filho, Alonso, Chagas, Szuster, & Sussekind, 2009).

Until the early 1980’s, investment in infrastructure projects to further economic and social development was traditionally undertaken by national governments and states. Changes in this model began as a consequence of the oil crisis of the 1970’s, which led to an increase in inflation and public debt in non-oil producing countries and reduced capacity to invest in infrastructure by governments. This process of reform resulted in an increase in the participation of the private sector in public infrastructure projects and in the trend to privatize state owned firms worldwide.

However, in all cases, private concessions are only possible when the project is economically sustainable and the operation is able to provide the required return on the investment made by the private partner. Most of the early concession projects fell into this category, as the most profitable and less risk ones were the first to be granted to the private sector. As this stock of high return projects decreased, there remained a significant number of projects of public interest, but which failed to attract private investments due to an expectation of lower returns or excessively high risk.

In Brazil, the rampant hyperinflation and severe economic crisis in the 1980’s created the necessity to redefine the government role, the privatization and concession program that took
place in the following decade. Only in 2004, the Brazilian government created a specific legislation for PPP projects (Federal Law nº 11.079 of December of 2004) which contemplated two major types of PPP: Sponsored (PPP Patrocinada), where a major part or all of the project’s revenues derive from tariffs collected from users of the service, thus subjecting the concessionaire to variations in demand and market risks; and Administrative (PPP Administrativa), where the main beneficiary of the services is the government, which in turn guarantees a fixed payment in lieu of project revenues for the contract duration. In this article we focus on the latter case.

For the private investor, a consistent valuation of a PPP project is the basis for shareholder value creation, and the Discounted Cash Flow model (DCF) is the traditionally valuation method used for this class of project. Nonetheless, the use of DCF methods for the valuation of PPP projects presents limitations, as it fails to capture the impact and value of uncertainties and managerial flexibility that may be embedded in these class of projects (Martins, Marques, & Cruz, 2013). These flexibilities, or options, may allow the firm to defer the start of the project, expand operations, alter the project in any way or even abandon the venture in response to changes in market conditions observed after the project is initiated in order to maximize returns or minimize losses. Given that these managerial flexibilities can be seen as options, their value can only be captured through the use of option pricing methods, such as the real options approach.

Infrastructure PPP projects are full of uncertainties, especially in the construction phase in which the completion risk is materialized as cost uncertainty, schedule delay and technical issues, that severely impact the future cash flow generation during the operational phase (Ho & Liu, 2002). In this case, there is always the risk that the project could be abandoned midway, particularly if the valuation was not adequately performed or if there are significant uncertainties over the capital investment amounts required for its full implementation.

In this study we develop a real option model to price the flexibility of investment combined with the abandonment option, depending on the capital expenditure (CAPEX) required to each phase of a PPP project. Considering the abandonment option is not desirable by the government due to Brazilian fiscal rules, we highlighted the relevance on established a level of contract penalty.

As an application, we analyze the deployment of a phased PPP project to build, operate and transfer (BOT) a broadband wide network for the state government of Rio de Janeiro, Brazil, where the capital expenditure of each phase are assumed to be uncorrelated due to the technical and geographical settings of each area of implementation. The government provides the firm with a fixed revenue payment in exchange for the investment, construction and operation of an infrastructure project, which is typical of an Administrative PPP in Brazil.

In this case, all the uncertainty is concentrated on the level of capital expenditures required to complete each phase, in a way that is similar to the case of R&D projects, where the time and investment costs to complete development of an innovative drug, a software program or product is uncertain. While this abandonment option is not explicitly granted in PPP contracts as the government has no interest that the project be discontinued, there is always the risk that the firm may default on its obligations. For this case, PPP contract typically have penalty clauses that act as a disincentive for this decision. In this study we addressed how to determine optimal levels of this penalty.

This article is organized as follows. After this introduction, we contextualized infrastructure investments using PPPs and it relevance to design broadband infrastructure in Brazil. In section
three we discussed the literature review focus on PPP feasibility analysis and real options. In section four we present the case of a broadband PPP project in Brazil, followed by a real options modeling for pricing flexibilities in phased PPP projects. In section six we present our results and finally we conclude.

2 Infrastructure investments and PPP in Brazil

The privatization process that occurred in Brazil the 1990s was focused on attracting private investment and simplifying the operation of the federal government. In 1998, the national privatization program had among its objectives the indirect execution of public services, through concessions, as the government redefined its role and delegated to the private sector activities in industry, infrastructure and services while it focused its activities to the regulation and supervision of public services. This phase lasted until the end of 2004 and included the privatization of large public enterprises, especially in telecommunications, steel, petrochemical, energy, mining and fertilizer industries (Silvestre, Hall, Matos, & Figueira, 2010).

Even after the privatization process, the infrastructure investments remained around 1% of GDP and the total gross investment was only 2.2% between 2002 and 2005 (Ferreira & Nascimento, 2006). In 2008 the Brazilian government invested only 2% of GDP in infrastructure projects, while other emerging countries had rates of 6.2% *(Chile), 5.8% (Colombia) and 5.6% (India) (Frischtak, 2013). The ratio of investment to GDP in Brazil is lower than the average of 3% among emerging economies in recent years. Figure 1 shows the ratio percentage of investments in infrastructure to Gross Domestic Product (GDP) of some emerging countries.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{infrastructure Investments as Percentage of GDP}
\caption{Infrastructure Investments as Percentage of GDP}
\end{figure}


According to Oliveira et al. (2013), the Brazilian infrastructure would require investments of 5% of GDP for a prolonged period in order to reach levels similar to the East Asia industrialized countries, which would depend on the joint efforts of the public and private sectors. In this sense, Public-Private Partnership (PPP) is an important alternative to enable infrastructure projects, presenting a series of advantages when compared to traditional concessions (Martins, Marques, & Cruz, 2014):

a) Setting goals and incentives to the compliance of service improvement;
b) Management transparency;
c) Innovation and best management practices;
d) Maximization of the service quality;
e) Optimization of risks allocation between public and private partners.

The Brazilian legislation took into account the international experience and tried to minimize failures, establishing explicit criteria for risks distribution and strict definitions to projects that can be met through PPPs. In 2004, the Brazilian federal law nº 11.079, defined the sponsored and the administrative PPP models. In the sponsored model, the remuneration of private partner comes from the users (citizens) that pay for all or for part of the public service offered. In the administrative model, the public partner pays to the private partner for the investment, even involving infrastructure investments or supply of goods and facilities.

Nowadays, there are many PPPs in Brazil organized by various governmental levels, as can be seen in Figure 2. Most of them concentrated in transport projects.

![Figure 2: Number of PPP Running Projects and Bidding Projects in Brazil](image)

Source: Oliveira, Marcato e Scanzufca (2013).

Although many efforts has been expended upon PPPs projects in Brazil, the use of PPP to design broadband infrastructure still an issue. In a recent qualitative research conducted in conjunction with the World Bank, Giambiagi and Pinheiro (2012) showed that the perception of Brazilian managers of private sector considered that the infrastructure gaps are really concentrated on transport restrictions, but at least 17% of the sample attributed to telecommunications and broadband infrastructure one the main Brazil’s infrastructure gaps.

According to OCDE (2008), broadband infrastructure impacts the market competitiveness, economic growth and there is a significant correlation between social development and broadband network penetration. It estimates that for each 10% increment in broadband penetration, the economic growth is impacted in 1.38%. In a similar fashion, the McKinsey & Company identified that for each 10% in broadband penetration, the country GDP can be range from 0.1% to 1.4%. Booz & Company also noticed that “10 percent higher broadband penetration in a specific year is correlated to 1.5 percent greater labor productivity growth over the following five years” (Friedrich, Roman, Sabbagh, El-Darwiche, & Singh, 2009).

The Building Broadband Report of World Bank (2010) observed that broadband networks are adding value to public services, providing automated services which were previously done manually, such as financial services, e-health care provision and public safety. But, the services to be offered depend on the construction of physical network.
There are two main types of broadband physical infrastructure: the access network and the backhaul/ core network. The access network provides connectivity between end users and network node, at which point the core network is connected with the access network. The backhaul/ core networks provide connectivity over large distances using links between network nodes, known as backbones. The core network can be used to connect cities across the country and the backhaul can connect local exchanges to core networks. Considering the large number of applications passing through the network, the choice of using optical fibre cable technology is important to forecast high capacity. But in mountain areas, for example it is also common to see high capacity microwave links, due to the geographical limitations to dig tranches to the backbone (ITU, 2012).

Investment decisions about the most appropriate infrastructure for broadband deployment are also related to government policy. The government authority must decide the aims of the broadband investment, defining socio-economic benefits, regional development, competition or the most basic needs for individuals, communities and business (Kim et al., 2010), as follows:

- **Private Design Build and Operate (private DBO) model** – public funding for a private sector organization to assist in the deploying of the network offering open wholesale access. The public sector has no role in the ownership or running of the network.

- **Public DBO model** – the public sector manage all aspects of the deployment and operation of the network. A network company is formed by the authority, and offers wholesale and services.

- **Public outsourcing model** – a private-sector organization is contracted to cover all aspects of the design or construction of the network. The public sector retains ownership and some control, but the infrastructure is built and operated by the private sector.

- **Joint venture model** – public and private sectors split the network ownership. Network construction and operation are undertaken by a private-sector organization.

Among the investment models, public and private BDO models are by far the most commonly used for PPP projects design. In broadband projects for public sector this model is prioritized due to the long term needs and contracts complexities. Working with the private sector can bring many advantages, including expertise and high efficiency in service level agreement (SLA) deliveries, helping to ensure the project sustainability. Nonetheless, the government authority should consider private investment from within and outside the telecommunication sector, which can also come from operators, utilities and equipment providers.

In a little over 10 years, the investment models deployment to the accessibility of broadband to consumers and business have become crucial for economic and social development. In developing countries, governments and regional authorities are still facing challenges of construction, operation and financing broadband infrastructure (WBG, 2013). The use of PPP framework has been increasingly adopted by many countries, allowing the reduction of government operational expenditures and increase the efficiency of the services provided.

Although in many countries there are a lag in broadband adoption, according to the World Bank (2010), countries in North America and European Union accounted for more than 50% of the world’s 1 billion mobile broadband subscriptions, while South Asia and African countries had less than 3% of the global participation. Not surprisingly, many Latin American countries recognize this issue and now are focused on implementing broadband infrastructure to promote economic development, market competition and to attract private investment. The correlation between urbanization, density population and broadband services in Latin America represents
another aspect to keep focus on broadband infrastructure development and a likely reason to include broadband as a government policy formulation.

In that context, the broadband implementation through public-private partnership (PPP) in Brazil can be seen as a collaborative approach to promote and later universalize broadband services. Nonetheless, this implementation requires huge investments in broadband infrastructure, which can be seen as the main uncertainty variable over the construction phase. Due to the irreversibility and time to build of this kind of investment, it is also important to provide a financial modeling that can be friendly addressed the uncertainty of the project. The real option model developed in this study derives the decision rules for this class of projects, subjected to irreversible investments.

3 PPP Feasibility Analysis and Real Options

The feasibility analysis of public private partnership (PPP) is one of the main critical factors for successful implementation of this sort of projects. The traditional discounted cash flow (DCF) is the most often used method to value the feasibility of PPP projects in infrastructure (Araki & Yoshizu, 2007; Daube, Vollrath, & Alfen, 2008; Graham & Harvey, 2001). This method is also frequently combined with the value for money approach which has been also considered relevant to value projects from the government’s perspective (Grimsey & Lewis, 2005; Ke, 2014; Regan, Smith, & Love, 2009). Although these methods are easily accepted by market specialists and government authorities, they fail to not incorporate uncertainties which can be crucial for infrastructure projects sustainability in the long term.

In order to deal with uncertainties embedded in large scale investments related to PPP contracts, real options (RO) method can provide a more consistency approach for financial valuation modeling for this class of projects. Rose (1998) was the first to apply real options to value flexibilities in a PPP contract. The author used Monte Carlo method to simulate the underlying uncertainties of a toll-road concession project in Australia, focused on the interaction effect between the contract government option to terminate the concession period and contract concessionaire option to defer the concession fee payment.

Ho and Liu (2002), developed an option pricing model to evaluate government debt guarantee of privatized infrastructure projects in a BOT (build operate and transfer) approach, considering construction costs and net cash flow as the main uncertainties of the project, which was resolved using binomial tree model. Bowe and Lee (2004) assessed expansion, postponement, reduction and abandonment options during the PPP contract horizon, using binomial lattice approach in a railway construction project, considering a BOT model of a high-speed train in Taiwan. Zhao, Sundararajan, and Tseng (2004) developed a multistage stochastic model for a highway development, considering traffic demand, land price and condition index at time, as the tree uncertainties of the model. The problem was solved applying an algorithm using Monte Carlo simulation and least-square regression. Garvin and Cheah (2004) examined a toll road project in the USA, using Monte Carlo simulation and binomial tree for pricing deferment options and emphasized the importance of real options instead of the traditional methods in infrastructure valuation projects.

Huang and Chou (2006) proposed a model involving a minimum revenue guarantee (MRG) combined with abandonment option in the preconstruction phase of the Taiwan High-Speed Rail Project, which was formulated as series of European put options. Cheah and Liu (2006) proposed a simplified model to analyzed the economic and financial feasibility of a Malaysia-Singapore second crossing, considering an expansion option with traffic guarantee using Monte Carlo
simulation. The author suggests that relevant items of concession contracts need to be treated as options. Alonso-Conde, Brown, and Rojo-Suarez (2007) analyzed the Melbourne Citylink toll road project as a case study to value financial incentives treated as real options offered by governments to the private partner in PPP agreements, which affects investments and the way of transferring value to the concessionaire. Chiara, Garvin, and Vecer (2007) suggested a model for quantifying the value of flexibilities in a BOT concessions contract, using Bermudas, European and Australian options. The valuation structure used the least-square Monte Carlo approach for optimal exercise decision upon estimating the continuation value.

The use of guarantees as options in PPP contracts continue to be observed in the recent literature. Brandão and Saraiva (2008) developed a more consistency model in evaluating the effectiveness of traffic guarantees in a toll road concession model to reduce the risks of projects and government costs in PPPs, proposing a real options modeling which limits the government's liabilities with a minimum traffic guarantee (MTG), as opposed to MRG that’s commonly seen in previously studies. In a similar fashion, Takashima, Yagi, and Takamori (2010) addressed a financial framework to analyze how contractual arrangements impact private and government partners dealing with investment cost and risk sharing in PPP projects. Rocha Armada, Pereira, and Rodrigues (2012) had also developed a pricing option model to value government incentive in PPP arrangements, such as investment subsidy, revenue subsidy, minimum demand guarantee, and a rescue option, that can be implemented to promote immediate investment and to compensate the private partner for the deferment option value.

Brandão et al. (2012) presented a real option model to evaluate risk reduction level that could be obtained by the company as well as the cost and the risk of these guarantees, which would be relevant to value toll road projects such as the Metro Line 4 of São Paulo subway system studied by the authors. Krüger (2012) developed a model using assumptions of the theory of incomplete contracts, through which they assess the implications of using contract options of public-private partnerships (PPPs) for the expansion of road infrastructure in Sweden. It examines government intervention to maximize the social benefit of the project, vis a vis the private interests and ownership impacts on timing of expansion options in PPPs contracts. Almassi (2013) proposed a finite-difference method based on continuous stochastic process to value contractual guarantee to assist governments in PPP projects. The exercise options strategies followed a multiple-exercise (Australian) guarantees structure. Chiara and Kokkaew (2013), developed a dynamic revenue insurance model for PPP contracts, providing a revenue risk coverage instrument as a government guarantee. This hedging instrument is modeled using Monte Carlo simulation as a multiple exercise boundary method and multiple least-square approach. Martins et al. (2014) identified different areas of real options application and contextualized the relevance of this methodology for PPP infrastructure arrangements, through a case study in an airport project using Monte Carlo simulation.

Although the large range of literature on the application of pricing techniques in infrastructure projects, the use of real options on pricing PPP contractual flexibilities still scarce. This study attempts to contribute to dissemination of real options methods on the structuring PPP contracts and to the investment uncertainty modeling, which can incentive the private partner to fulfill the contract.

4 The Project

In 2010 the Brazilian Federal Government launched the National Broadband Plan (PNBL), which aims to ensure the availability of broadband connection to low-income households and set
the target to triple broadband availability by 2015. The Federal Government has also urged state
governments to invest in broadband infrastructure, seeking to improve services for citizens and
business through e-government initiatives. Broadband penetration in Brazil is slightly higher than
the Latin American average, but lower than the Chilean, Argentinean and Uruguayan
penetrations. Despite its relativity good international connectivity, broadband accessibility in the
country is still scarce and rates are higher than the ones observed in North America and Europe.

In the end of 2013, the state government of Rio de Janeiro received project suggestions
carried out by private parties, also known as the manifestation of interest of private initiative
(MIP) proposing the development of a “Digital State” through the investment in infrastructure,
equipment and operations management, supported by the services provision of broadband
connection and IP platform technology. The aim of this proposed study was to connect multiple
state departments and agencies, allowing digital versions of the traditional public services to be
offered to the general population as well as improving the management of public resources. The
project intended to provide high quality connections and to reduce the cost of public e-services.

The State government chose to bid out a project to build and operate a broadband
infrastructure through a public-private partnership (PPP), including project implementation,
operation and maintenance of the broadband network, equipment, hardware, licenses, basic
infrastructure and spares for the operation a new integrated solution to the state government
administration. The primary purposed was to replace the current contracted firm that provides
internet network and telephone services at high annual costs that would gradually be substituted
for a new platform that integrates voice, data, video and images. This platform is to ensure
connectivity between sectors of the public administration through a Wide Area Network (WAN)
infrastructure of high capacity, complemented by metropolitan high-speed networks, not only
able to support the current needs of administrative and operational activities but also supporting
the implementation of future applications that could make use of new features of the network,
such as classification and traffic prioritization, improving efficiency and services productivity.

The network design and financing model was contracted out to a University, which used
computational tools based on topography and morphology geo-referenced maps and the input
parameters considering database of location (geo-referenced maps), where the network will be
deployed, the network location of users, traffic matrix between users and performance and quality
parameters of the network. The network will provide a hierarchical structure, consisting of a
backbone, backhaul and metropolitan networks. The main backbone interconnects the entire
metropolitan network and allows connection of the broadband network to other local networks,
telecommunications companies and to the international network.

The model and plan developed for infrastructure deployment was expected to take place for
five years. Each year of construction will represent a distinct phase of the project. Services will
be performed according to each phase of deployment, and the payment for services will be
conducted monthly over the 25 year life of the project. The phases of project implementation and
priority areas were defined considering the population of Rio de Janeiro, initially serving regions
with higher density. Then, given the recommendations of the State government, the project
phases also prioritized higher GDP concentration regions.

The total capital investment for the broadband project infrastructure deployment for the
State of Rio de Janeiro was estimated in US$574.91 million, considering the operational center
development, the fiber optic of the backbone and the metropolitan areas installation and
infrastructure works, which represents approximately 80% of network deployment. This
investment was based on technical studies and to the Request for Information (RFI) process with
representative infrastructure and telecom companies that already operate in the Brazilian market. Depreciation was calculated linearly and proportionately to the phase of the project, according to machineries and equipment demanded and complying with the concession contract deadline. The operational expenses represent approximately 11% of revenues, or US$6,52 million yearly and the income taxes represents 32.8% of the EBIT on average, according to the Brazilian tax legislation.

The public annual compensation to support the infrastructure deployment and the services provision is proportional to the infrastructure phases and to the range of services delivered. The concessionaire will be remunerated gradually during the construction and operational phase as shown in Table 1.

<table>
<thead>
<tr>
<th>Year</th>
<th>% of maximum compensation</th>
<th>% accumulated compensation</th>
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<tbody>
<tr>
<td>1</td>
<td>27%</td>
<td>27%</td>
</tr>
<tr>
<td>2</td>
<td>15%</td>
<td>42%</td>
</tr>
<tr>
<td>3</td>
<td>28%</td>
<td>70%</td>
</tr>
<tr>
<td>4</td>
<td>15%</td>
<td>85%</td>
</tr>
<tr>
<td>5</td>
<td>15%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Under the concession contract, once the private partner agree to government conditions and sign the contract, the project must be implemented. To reinforce the construction obligation, the government has defined a penalty of 10% of the capital investment (US$56,52 million) if the concessionaire decide to terminate the project.

The total public annual compensation is estimated to occur at the end of the fifth year, which is the beginning of the operational phase for the concessionaire. Table 2 shows a summary of the main characteristics of project base case.

<table>
<thead>
<tr>
<th>Table 2: Project parameters</th>
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<tbody>
<tr>
<td>Maximum public annual compensation</td>
</tr>
<tr>
<td>CAPEX</td>
</tr>
<tr>
<td>COGS</td>
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<tr>
<td>Project design</td>
</tr>
<tr>
<td>Penalty</td>
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<tr>
<td>WACC</td>
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<td>Risk-free rate</td>
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<td>Tax rate</td>
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<td>Inflation (per year)</td>
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</table>
5 The Model

The project is structured chronologically in three distinct phases: the planning phase, the construction phase and the operation phase. The project timeline is depicted in Figure 3.

![Figure 3 – Time line of the project](image)

In the planning phase, the private partner will have one year to develop the actual broadband infrastructure designs and blueprints. There will be no capital expenditures during the planning phase and all costs associated with it will be concentrated at \( t = -1 \) and will be equal to a fixed amount \( C \), i.e., there is no uncertainty with regards to the costs of designing the engineering project.

The construction phase begins at \( t = 0 \) and is expected to last \( k \) years. The construction of the broadband telecommunications network will take place in “sub-phases”, which will be completed sequentially throughout the \( k \) years. Each sub-phase lasts one year\(^1\). At the beginning of each construction sub-phase \( j \), i.e., at \( t = j - 1 \), the contractor incurs a capital expenditure \( I_j \).

The value of this cash flow is random at all times \( t; t < (j - 1) \), but is known with certainty at \( t = (j - 1) \). Following McDonald and Siegel (1986), we model this random cash flow as a random variable following a geometric Brownian motion:

\[
dI_j / I_j = \mu dt + \sigma dz
\]

(1)

The drift \( \mu \) and volatility \( \sigma \) in (1) are constant in each \( t \)-period but may change from one period to the next, reflecting different technical aspects of the engineering project in each sub-phase. Moreover, each of the \( k \) CAPEX cash flows is statistically independent from each other.

The capital expenditure \( I_j \) required in the respective sub-phase is applied gradually, according to each sub-phase schedule. Considering the uncertainty surrounding the actual invested amount in each sub-phase, we model the risk perception of the dealer as an abandonment option, which in this case means that it would not keep investing if the required investment exceeded its financial capabilities. While this abandonment option is not explicitly stated in the PPP contract as the government has no interest that the project be discontinued, in this study we focused on the value of the penalty to be imposed to discourage the project termination, a decision which, effectively, functions as an abandonment option.

Hence, at each time \( j - 1 \), before undertaking the required capital expenditure \( I_j \), only known at that time, depending on the risk perception the concessionaire could choose to discontinue the project and pay a fee \( F_j \) to the public partner. However, this is not a desirable outcome due to the social and economical importance of the PPP project.

Let \( V_i \) be the project value during the construction phase. The decision-making rule during the construction phase can be represented as in (2):

\[^1\) It is possible to model the optimal timing and durations of the construction schedule. However, given that the main uncertainties of the project revolve around the amounts of the capital expenditure themselves, and not their timing, we chose to consider a fixed construction schedule with random capital expenditures.\]
\[ V_t = \max \left\{ -F_t - I_t + \frac{E[V_{t+1}]}{1+r}, 0 \right\}, \forall t \in \{0,1,\ldots,k-1\} \]  

(2)

where \( r \) is the risk neutral discount rate appropriate to the project with options and \( E[V_{t+1}]/(1+r) \) is the expected discounted project value at the beginning of the next period. The meaning of equation (2) is straightforward: at each time \( t, \forall t \in \{0,1,\ldots,k-1\} \), the concessionaire will compare the value of the penalty fee \( F_t \) with the net present value of the future cash flows, represented by \( -I_t + E[V_{t+1}]/(1+r) \). The private partner will keep investing only if the NPV is greater than the abandonment fee. It is noteworthy to realize that the NPV can reach negative values, \textit{i.e.}, even if the subproject NPV is negative, it would still be economically feasible to keep investing as the financial loss would be higher if the firm chose to abandon the project.

After some algebraic manipulation, we rewrite (2) as follows:

\[ V_t = \max \left\{ F_t - I_t + \frac{E[V_{t+1}]}{1+r}, 0 \right\} - F_t = \left( F_t - I_t + \frac{E[V_{t+1}]}{1+r} \right)^+ - F_t \]  

(3)

From (3) it can be reasoned that the project value during the construction phase is equivalent to the sum of the payoff of a call option with the underlying asset equal to the random variable \( F_t - I_t + \frac{E[V_{t+1}]}{1+r} \) and the strike price equal to 0, and a fixed cash flow \( F_t \). Due to the sequential nature of the investing decisions in the construction phase, we model this phase of the project as a compound option (Dixit & Pindyck, 1994) as shown in Figure 4.

Moreover, at \( t = -1 \), the firm must decide whether to enter the bid process or not. Similarly, the firm will only take part in the competitive bidding if the expected discounted project value is greater than the proposal design cost \( C \).

\[ V_{-1} = \max \left\{ \frac{E[V_0]}{1+r} - C, 0 \right\} = \left( \frac{E[V_0]}{1+r} - C \right)^+ \]  

(4)

The payoff in (4) corresponds to the one of a call option exactly. In this instance, the underlying random variable is \( E[V_0]/(1+r) \) and the exercise value is the design cost \( C \).

\[ \text{Figure 4: The compound option to invest.} \]
Finally, the \((T - k)\) years after the construction phase will be devoted to operating the broadband network and providing broadband services to the state government. There is no uncertainty in this phase as the private partner is entitled to a fixed annual payment for having completed the construction of the network infrastructure.

6 Results

The traditional discounted cash flow (DCF) analysis indicates that the broadband infrastructure PPP project is economically viable. The net present value (NPV) of the project is US$28,74 million and the total required investment is US$574,78 million. The annual internal rate of return (IRR) is 7.39%, which is greater than the risk free rate of 6.8%, though it is lower than the estimated weighted-average cost of capital (WACC) that is equal to 10.39%. This shows that the project economic viability would probably be contested by the private partner, considering the large initial capital investment and the uncertainty revolving around the construction phase. The building uncertainties associated with the reinvestment obligation in the long term could represent a disincentive to the private partner to honor the 25-year contract.

When the possibility of abandoning the project is considered, we calculated the value of this real option to be US$9.09 million, representing an increase of 31.6% over the base case NPV. These calculations confirm that investing in the broadband infrastructure project is warranted.

A sensitivity analysis of the real option value was performed and its main results are illustrated in Figure 5. The option value increases monotonically with the volatility of cash flows and decreases monotonically with the value of the penalty fee. The first result was expected as the value of any option increases with uncertainty. The intuition behind the second result is that the lower the penalty fee, the easier it will be for the private partner to abandon the project in those scenarios in which construction costs turn out to be higher than expected. As the value of the penalty fee increases, it may not be economically sound for the private partner to abandon the project and it may choose to keep investing instead, even though the investment might be made at a loss.

![Figure 5: Sensitivity analysis of the real option value](image-url)
We also analyzed the sensitivity of the probability to abandon the project at $t = 0$ with respect to the penalty fee and to the cash flow volatility. These results can be found in Figure 6. Under the base case scenario, there is a 12.5% chance that the private firm will abandon the project at $t = 0$, with a baseline penalty fee of US$57.4 million.

In all volatility scenarios, the probability to abandon is monotonically decreasing with respect to the penalty fee. Once again, the intuition of this result is that as the penalty fee increases, the likelihood that the future NPV will be even smaller decreases as well, rendering the abandonment unlikely. In other words, the firm would only abandon the project at the high penalty fee scenarios if future cash flows are unexpectedly negative and large, a situation that corresponds to rare events from a statistical point of view.

Analyzing the results across the volatility dimension, it can be noted that the probability to abandon is equal to zero when there is no uncertainty around the CAPEX cash flows. This is expected as the base case NPV is positive (US$28.74 million) and the firm should invest in the PPP project if there is no uncertainty associated with the project’s cash flows. However, as uncertainty increases, so does the probability to abandon, even though the option value increases with greater CAPEX volatility, which, in turn, increases the overall value of the project.

It is interesting to note that the probability to abandon does not increase monotonically with respect to the volatility of cash flows. When the penalty fee is low, or even zero, the probability to abandon is a concave function of volatility, i.e., the probability to abandon initially increases with volatility and then reaches an optimal point from which it starts to decrease with further increases in volatility. We interpret this result as follows. At low penalty fees, an increase in volatility increases the likelihood that the firm will abandon the project as it is “cheap” to do so in case of extremely high realized CAPEX requirements, even though the option value increases. As volatility increases further, the value of the option itself starts to dominate and the final effect brings the probability to abandon down with higher levels of volatility. Hence, there seems to be a trade-off between the exit cost (penalty fee) and the potential future positive cash flows from the project, measured indirectly by the option value.

From a policy-making standpoint, the results displayed on Figure 6 present an answer to the following question: How high should the penalty fee be in order that the probability to abandon is equal to $x\%$ at a given level (or range) of cash flow volatility? This tool allows policy-makers to set penalty fees in a way that these fees do not discourage private investment and, at the same time, do not encourage empty investment proposals.
Conclusions

We developed a real option model to value the flexibility of sequential investments associated with an imbedded option to abandon the project in multiple occasions. We applied the model to value the economic viability of a broadband infrastructure project via a public-private partnership (PPP) arrangement in Brazil. This flexibility arises from the uncertainty surrounding the required capital expenditure (CAPEX) in each phase of the PPP project.

Our results suggest that the PPP bidding process can be forfeited by investors who assess project value using only the traditional discounted cash flow (DCF) approach. To investigate this issue further, we employed a real options model that considers annual capital expenditures as the main source of uncertainty in the PPP project. We modeled each CAPEX cash flow as independent geometric Brownian motion (GBM) stochastic processes and calculated the value of sequential abandonment options.

We calculated the value of the real option to be US$9.09 million, which represents up to a 31% increase over the value of the baseline project obtained by the DCF analysis. We also simulated the probability to abandon the project as a function of the penalty fee and CAPEX volatility. The baseline probability to abandon is 12.5% in the first year and it decreases substantially in the next four years of the construction phase; it is less than 0.5% in the 2nd year and less than 0.7% in the 3rd year. The probability to abandon the project at years 4 and 5 are virtually zero.

Our major contribution is the development of a quantitative tool that allows governmental policy-makers to design PPP projects with the most appropriate penalty fee schedule. The balanced setting of abandonment fees can be crucial in a PPP project: set them too high and they may scare private investors away; set them too low and private investors may abandon the project more often than desired.

The conclusions of this study are limited by a few assumptions. Firstly, we assumed that the concessionaire would be entitled to a fixed annual payment during the operational phase, ignoring any possibility of future contract amendments. We also assumed that the relevant project uncertainties are concentrated around each annual CAPEX investment during the construction phase, without any consideration regarding future technological advances or upgrades that will
probably take place over the tenure of the project. Additional sources of uncertainty such as changes in operational costs and equipment efficiency were not considered either as their economic impact is relatively minor. We did not look into the socio-economic impact of this project in a developing region of a country like Brazil.

8 References


