Correlation-Based Systemic Risk Measures and Stock Market Regimes☆

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Abstract

This paper exhibits some systemic risk measures based in asset prices and their covariance. Using a Markov Switching model with time-varying transition probabilities, in-sample results shows that the covariance-based systemic risk indicators are useful to analyse crisis periods. Using a Herfindahl-Hirschman Index of principal components of he covariance matrix of residuals from a GARCH model is the most recommended systemic risk tracking tested in this paper when considering three regimes for the stock market from 1998 to 2013.

Keywords: Econophysics, Eigenvalue Entropy, Systemic Risk
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1. Introduction

A financial crisis may be troublesome for everyone in society. The economic panorama after in the post-2008 subprime crisis is not bright as may be exemplified by the Eurocrisis [1, 2, 3], huge governmental debt [4] and political radicalism [5]. If there is a place that seems very calm, surprisingly, that is the US stock market. In figure 1, it is possible to see that the S&P500 index is growing in an apparently sustained way since the bottom of the crisis in 2008.

This paper uses a simple systemic risk measures to assess what is happening in the US stock market: the absorption ratio by [6]. Previous literature — the first paper of this dissertation — shows this measure Granger-causing the VIX index, but they do not have a strongly significant impact (p-value ¿ 0.01) in returns after 2002. The interpretation proposed here is to understand the Absorption Ratio and the Herfindahl-Hirschman Index as a concentration measure of the risk factors estimated using the principal component analysis and understand how does it affects the stock market behaviour.

Absorption Ratio is a well-known measure discussed in the survey by [7]. Its main idea is to use the concentration of risk in the largest eigenvalues of a given portfolio to assess the systemic risk. It is built using the covariance matrix. This indicator was also used for funds returns by [8] The eigenvalue entropy was applied in the econophysics literature [9, 10] using correlation of assets. They are examples of what can be considered correlation-based systemic risk indicators.

Systemic Risk literature, as described in [7], is very rich in indicators. There are measures based in macroeconomic analysis directly [11, 12]. These indicators are responsible for understand if the macroeconomic condition is prone to systemic collapses. [13] concluded that credit and monetary contractions are the most useful series to analyse the financial environment. It is possible to figure that the objective of these indicators is different from a simple risk concentration/dispersion tool.

A more similar tool are based in network effects [14, 15]. These measures are related to contagion risk and how much a node of a network can impact in other nodes. For instance, [8] created a network

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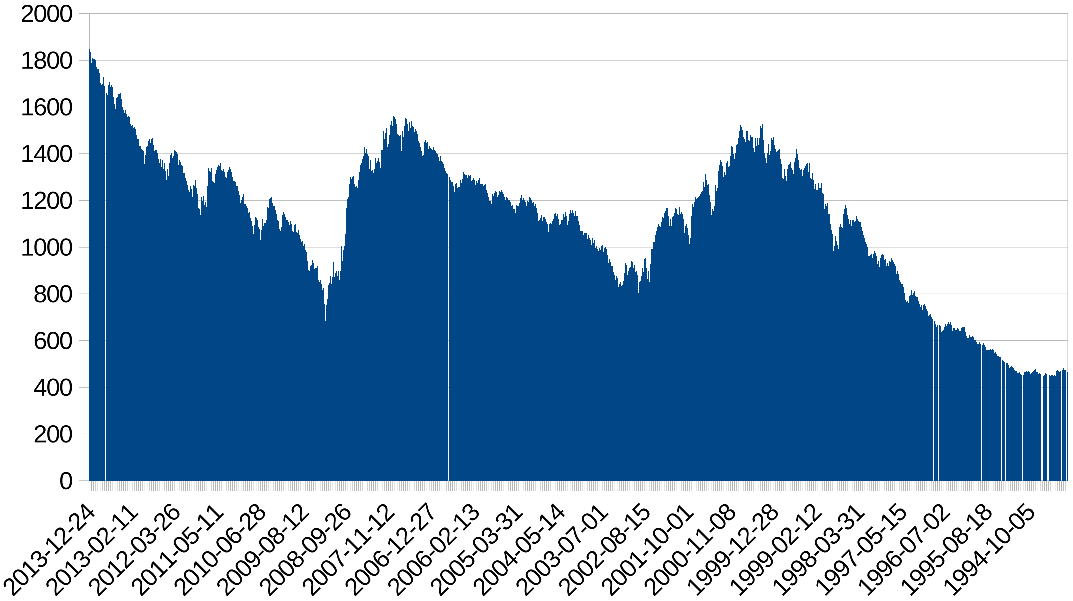
Figure 1: The S&P500 over time.



of Granger-causation between returns of hedge funds, brokers, insurance companies and banks. Other approaches are based in network econometrics [16]. [17] developed an algorithm based in the LASSO estimator that considers both contemporary and long run correlations.

Network analysis is also studied in econophysics. This literature usually considers the correlation matrix and partial correlations [18, 19, 20]. Techniques as random matrix theory [21, 22] are used to detect abnormal patterns in the eigenvalues derived from correlations. The usage of this technique applied to Absorption Ratio is developed in this paper and a novel indicator, the filtered Absorption Ratio (fAR) is built. A random matrix theory application is used to check how the eigenvalues used in the correlation matrix are different from the whole set of eigenvalues of financial data. This analysis leads to the conclusion of overestimation of risk factors in traditional Absorption Ratio indicator. While [6] recommends 70 eigenvalues for the dataset used here, an indicator using 15 filtered eigenvalues is built in this application.

The procedure of filtering eigenvalues is to simply use a historical volatility model to fit the returns before estimating the covariance matrix. The residuals are bootstrapped to generate new random covariance matrices. The reasons for this procedure are related to the literature on Value-at-Risk, especially [23, 24], and are best described in section 3. The traditional indicators are presented in section 2.

A more complex analysis is built using the Markov Switching method proposed by [25, 26]. The returns are analysed using a time-varying transition probability matrix, being the probabilities function of the indicators. Implementations using two and three states are built and the model that better fits the data is the filtered Herfindahl-Hirschman with 3 states.

Qualitatively, the behaviour of the indicator is not very different from the model with fixed transition probabilities. However, a more deep backtesting shows that adding a systemic risk model to the transition probabilities leads to a more stable prediction of bad economic moments. This test was built with the filtered Absorption Ratio to show that adding a model — even one that has ambiguous results (using AIC and BIC) compared to the time-fixed transition probabilities — can lead to a better prediction. This analysis is developed in section 4.

2. Principal Components and Risk Factors

In industrial organization, two measures of analysis are well know and represent an initial analysis of any market structure: concentration rate and the Herfindahl-Hirschman Index (HHI) [27, 28]. In finance, [6] discusses some recent measures are analogous to them.

The Absorption Ratio is built using the eigenvalues from the covariance matrix and the Eigenvalue Entropies are built using correlation matrices. In the first paper of this dissertation, was shown that the Shannon Eigenvalue Entropy has similar properties of Rényi Eigenvalue Entropy, that is an immediate transformation of the HHI index. In this paper, the measure considered is the Absorption Ratio and its HHI considered in [6].

The measures may be constructed with normalized eigenvalues:

$$\hat{\lambda} = \frac{\lambda}{tr(\Sigma)} = \frac{\lambda}{N} \tag{1}$$

where N is the number of assets analysed, λ is an eigenvalue and Σ is the matrix analysed — covariance for absorption ratio and correlation for eigenvalue entropy. The formula for absorption ratio is simply:

$$AR = \sum^{N/5} \hat{\lambda}_i \tag{2}$$

where N is the number of assets used to calculate the index. Another indicator suggested is the standardized shift in the absorption ratio, constructed using the difference between the short term and the long term averages of the Absorption Ratio:

$$AR_{shift} = \frac{(\overline{AR}_{\text{15 Days}} - \overline{AR}_{\text{1 Year}})}{\sigma_{AR_{\text{1 year}}}} \tag{3}$$

The Herfindahl-Hirschman index of the eigenvalues may be written as:

$$HHI = \sum^{N} \hat{\lambda}_i^2 \tag{4}$$

This index was tested — when calculated using the covariance matrix estimated with 500 trading days — in [6]. This indicator was shown to perform worst than the Absorption Ratio.

The last indicator is a hybrid between the AR_{shift} and the HHI. It is simply:

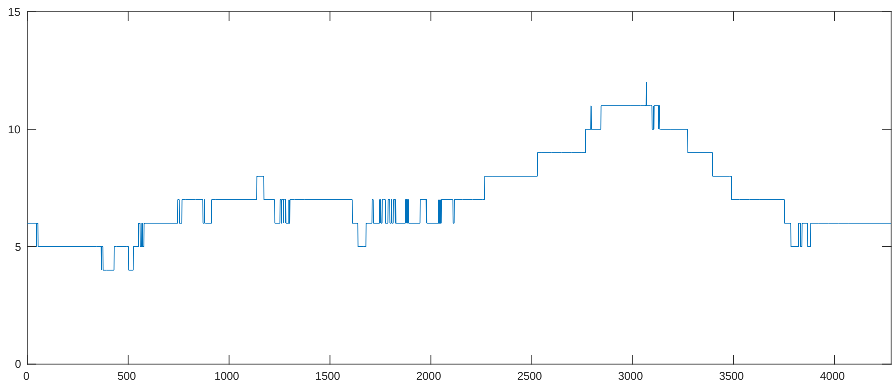
$$HHI_{shift} = \frac{(\overline{HHI}_{\text{15 Days}} - \overline{HHI}_{\text{1 Year}})}{\sigma_{HHI_{\text{1 year}}}} \tag{5}$$

3. Filtered Concentration

A novel systemic risk indicator based in the Random Matrix Theory is proposed in this section. Random Matrix Theory is based in the distribution of eigenvalues of a random matrix with normal observations. If the dimension of the analysed random matrix is TxN and $T, N \to \infty$, where $Q = \frac{T}{N}$ is greater than one and is finite, then this distribution, also know as Marcenku-Pastur is:

$$\begin{aligned} \rho(\lambda) &= \frac{Q}{2\pi\sigma^2} \frac{\sqrt{\lambda_{max} - \lambda}\sqrt{\lambda - \lambda_{min}}}{\lambda} \\ \lambda_{min}^{max} &= \sigma^2(1 \pm \sqrt{\frac{1}{Q}}) \end{aligned} \tag{6}$$

Figure 2: The number of non-noisy eigenvalues over time, using as criterion $\lambda_{it} < \lambda^{cov}_{max}$.



This distribution provides a theoretical framework to work with, but as it is possible to see, Q is not designed to the approach using covariances directly and it is based in normal distributions. In order to overcome these issues, a quasi-Monte Carlo simulation of the stock returns is used. For each stock series an AR(1)+GARCH(1,1) [29] is estimated and the parameters and residuals are stored. They are used to generate new series by bootstraping the residuals, in a process inspired in the Filtered Historical Simulation of Value-at-Risk [23, 24]. The reason for adopting this process is to filter the individual and predictable risk from the systemic risk analysis. These new series are used to create 10.000 new matrices 500x346 and calculate an empirical distribution of eigenvalues.

The data used in this paper was taken from Yahoo! Finance and its composed by all stock prices — 346 firms — from the S&P500 (collected August 31st 2014) from 1994 to 2013 available without missing observations. After calculating the AR_{shift}, this data is from 1996 to 2013. The estimated Random Matrix Theory parameters are $\lambda^{cov}_{min} = 0.0244$ and $\lambda^{cov}_{max} = 3.5637$.

These parameters are used for estimating the optimal n for the traditional Absorption Ratio (tAR) and for checking if the majority of them are really useful compared to the eigenvalues eliminated. This is exhibited in figure 2. This figure shows the optimal n using the λ^{cov} estimated.

This analysis suggests that the current number of eigenvalues used to calculate the Absorption is excessive. The procedure using $n \approx N/5$ may, therefore, add some noise to the estimative. However, to avoid jumps by accepting a different number of eigenvalues — and considering that there is some estimation error in λ^{cov}_{max} — in fAR there is a fixed number of 15 eigenvalues. Figure 3 shows how the filtered Absorption Ratio (fAR) captures a different behaviour than tAR.

Two simple metrics are useful to compare if the impact of noisy eigenvalues in the Absorption Ratio is smaller than in the HHI. The metric for the Absorption Ratio is:

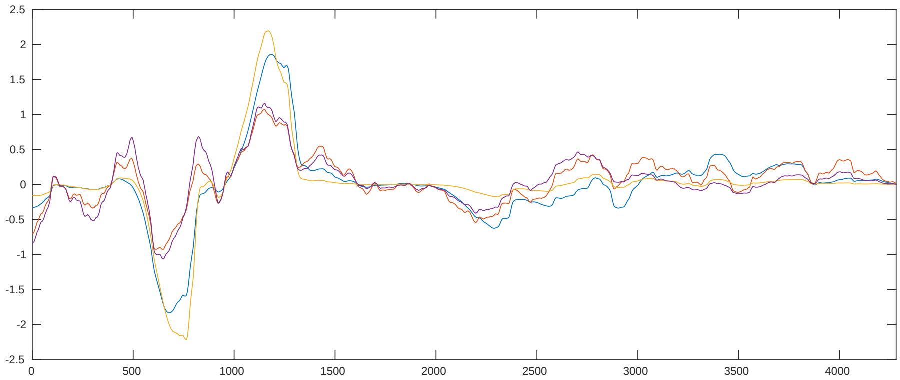
$$N_{AR} = \frac{\sum_{i>a}^{n} \lambda_i}{\sum_{i=1}^{n} \lambda_i}, \lambda_a > \lambda_{max} \tag{7}$$

The metric for noise in the HHI is analogous and defined as:

$$N_{HHI} = \frac{\sum_{i>a}^{N} \hat{\lambda}_i^2}{\sum_{i=1}^{N} \hat{\lambda}_i^2}, \lambda_a > \lambda_{max} \tag{8}$$

These metrics can be used for any kind of eigenvalue estimation, as the ones based in covariances and the ones based in correlations. The comparison of the two metrics allows to verify how much the eigenvalues that survived the RMT analysis influence the whole behaviour of the indicator. Their comparison

Figure 3: The blue line is the value of tAR, the red line is the value of fAR, the yellow line is the value of the HHI and the purple line is the value of filtered HHI.



for λ in figure 4 shows the impact of noisy eigenvalues is smaller in the Henfindahl-Hirschman Index than in the Absorption Ratio.

4. Systemic Risk Impact on Returns

This section deals with the estimation of a Markov Switching model with time-varying transition probabilities. The transition probabilities are a function of the systemic risk measures. They were estimated using the MATLAB toolbox developed by [30]. The estimation method is described in [26].

The model is composed by a simple regression of:

$$y_t = \alpha_S + \varepsilon_t; \varepsilon \sim \mathcal{N}(0, \sigma_{St}) \tag{9}$$

The parameters are regime-dependent (S). The regime follows a Markov Chain with transition probability:

$$\begin{bmatrix} q_{11} & q_{12} \\ 1 - q_{11} & 1 - q_{12} \end{bmatrix}, \tag{10}$$

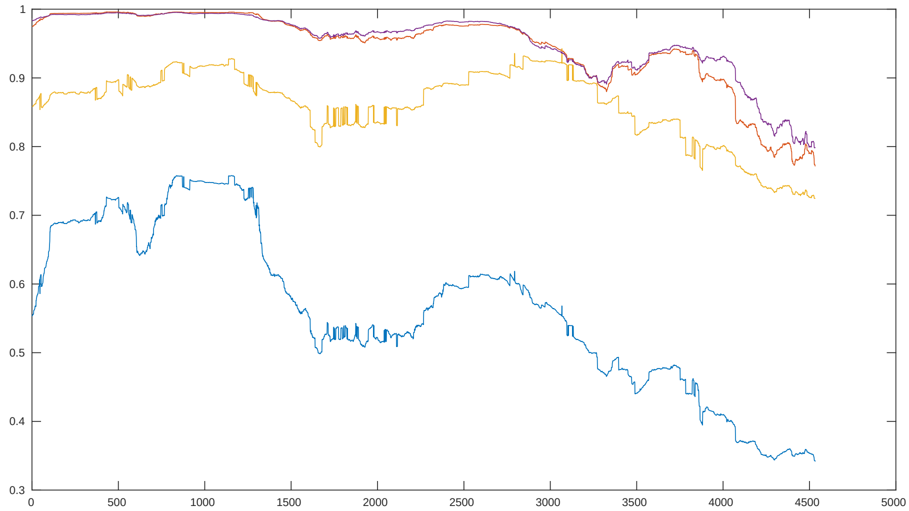
for models with two states and

$$\begin{bmatrix} q_{11} & q_{12} \\ (1 - q_{11})q_{21} & (1 - q_{12})q_{22} \\ (1 - q_{11})(1 - q_{21}) & (1 - q_{12})(1 - q_{22}) \end{bmatrix} \tag{11}$$

for models with three states. q_{ij} is built by a cumulative normal distribution of the parameters estimated.

Five models were tested: the Markov Switching with fixed transition probabilities, the time-varying transitions probability with shifts of the Absorption Ratio and the filtered Absorption Ratio, the traditional HHI and the filtered HHI. The initial estimation is using two regimes. Qualitatively, there is not any great difference in the behaviour of the models. Using the AIC and the BIC, the filtered models outperformed the noisy ones and the absorption ratio were better than the HHI indexes. Overall, the best indicator was the filtered absorption ratio, as shown in table 1.

Figure 4: The blue line is the distortion of tAR, the red line is the distortion of HHI, the yellow line is the value for fAR and the purple line is the value of filtered HHI. It is clear that the Absorption Ratios have more noise than Herfindahl-Hirschman Indexes

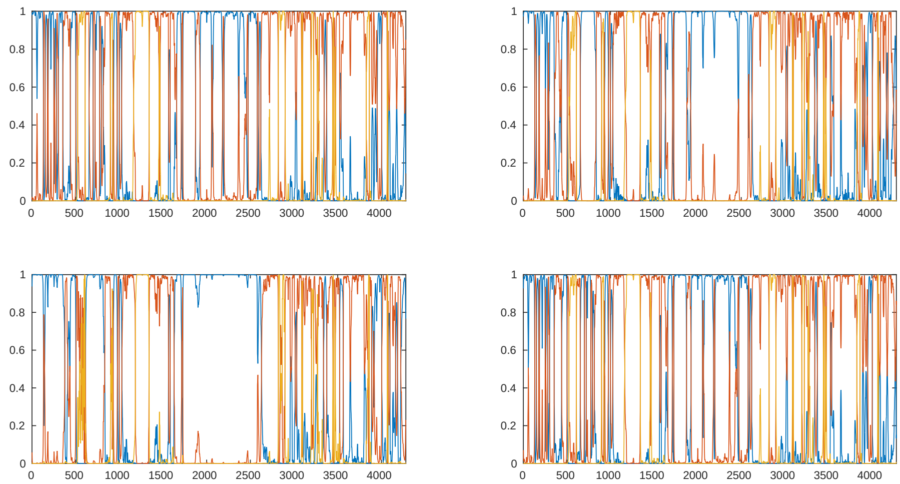


	Null	tAR	fAR	tHHI	fHHI
Mean(1)	0.00068**	0.00072**	0.00070**	0.00069**	0.00068**
Mean(2)	-0.00102▲	-0.00102▲	-0.00094	-0.00102▲	-0.00099▲
Variance(1)	0.00007**	0.00007**	0.00007**	0.00007**	0.00007**
Variance(2)	0.00042**	0.00041**	0.00040**	0.00042**	0.00042**
q(1,1)					
intercept	2.38702**	2.30245**	2.43136**	2.36852**	2.41771**
slope		-1.26944**	-1.72267**	-0.53695**	-0.91671**
q(1,2)					
intercept	-2.015817**	-1.84660**	-1.87463**	-1.93864**	-1.90950**
slope		0.17833	0.28477	-0.13967	-0.32806
AIC	-26425.76	-26444.89	-26448.69	-26433.21	-26439.09
BIC	-26387.59	-26394.00	-26397.80	-26382.32	-26388.20

More generally, it seems that the systemic risk indexes are not significant to analyse the probability of leaving the bull market. The model is prone to some asymmetry, revealing the crisis but not all regimes. A further investigation, imposing three regimes,

Imposing three regimes, the model incorporating the Absorption Ratio and the filtered Absorption Ratio is more stable. Generally the behaviour follows the same as the null but with less variations, as shown in figure 5 for AR, fAR and the filtered Herfindahl-Hirschman Index. For all models, there is the imposition of an intermediary stage: for instance, it is impossible to leave stage 1 and go to stage 3 and vice-versa. The results are in table 2.

Figure 5: The smoothed probabilities of each stage. The left upper graph is the model with fixed transition probabilities, the left down is the model with fAR, the right upper is the model with tAR and the right down is the model with fHHI.



	Null	tAR	fAR	tHHI	fHHI
Mean(1)	0.00104**	0.00091**	0.00080**	0.00093**	0.00104**
Mean(2)	0.00000	-0.00002	0.00003	-0.00001	0.00000
Mean(3)	-0.00145	-0.00166	-0.00236	-0.00160	-0.00145
Variance(1)	0.00004**	0.00004**	0.00005**	0.00004**	0.00004**
Variance(2)	0.00013**	0.00015**	0.00016**	0.00014**	0.00013**
Variance(3)	0.00072**	0.00075**	0.00090**	0.00744**	0.00072**
p(1,1)					
intercept	2.01298**	1.99101**	2.33394**	2.09014**	1.95943**
slope		-2.02145**	-2.28525**	-1.70435*	-0.47347▲
p(1,2)					
intercept	-2.17118**	-2.09103**	-2.05819**	-2.17699**	-2.12630**
slope		-0.12725	-0.68539	-0.20145	-0.72973**
p(2,2)					
intercept	2.46963**	2.45246**	2.02849**	2.44296**	2.49428**
slope		-0.14199	1.66533**	-0.06620	-0.13240
p(2,3)					
intercept	-1.88472**	-1.79758**	-1.35252**	-1.80046**	-1.82421**
slope		-0.31582	-1.34837**	-0.47300▲	-0.63614*
AIC	-26754.76	-26758.48	-26766.38	-26750.38	-26771.67
BIC	-26678.42	-26669.41	-26677.32	-26661.32	-26682.60

The model with lower AIC was the filtered HHI Index. It is clear that the traditional HHI and the traditional AR performed worst than their filtered counterparts. From figure 5, it seems that the fAR captured better stylized facts, such as the great stability from 2002 to 2006 and the financial crisis period, however the in-sample fit is higher for the fHHI model.

5. Conclusion

In this paper the Absorption Ratio and the Herfindahl-Hirschman Index of Principal Components are analysed as tools for systemic risk management. The first conclusion is that, in the actual form, Absorption Ratio considers too many eigenvalues. The number fell from 70 to 15 in our dataset. The second conclusion is that using an indicator that considers the GARCH effects and the real distribution from data generates richer dynamics.

The new indicators, the filtered Absorption Ratio and the filtered Herfindahl-Hirschman Index, were used in a Markov Switching with time-varying transition probabilities model. The model with both lower Akaike Information Criterion and lower Bayesian Information Criterion was the filtered Herfindahl-Hirschman Index with three regimes.

The usage of GARCH models to filter predictable shocks and the criteria from Random Matrix Theory are useful to select the principal components for the models. It is important to notice that Random Matrix Theory presents a simple way to select the number of components to financial applications and that correlation-based systemic risk factors may be condensed by its usage. The usefulness of this approach is clear in the Markov Switching tests proposed here.

Future research may include a forecasting test. Through forecasting tests using the crisis window — from 2007 to 2009 —, a rolling window may be estimated to analyse how the indicators predictions work in a moment of high financial instability. This is an important test for practitioners and the out-of-sample performance shall be analysed in the near future.

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