

## **Real Options with Bounded Rationality**

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We develop real options theory for real organizations under bounded rationality. Managers make decisions based on beliefs with information imprecision. Subjective estimates may suffer from behavioral biases (pessimism or overconfidence) and organizations adjust partially to new observations clinging to prior beliefs. Information imprecision, cognitive biases and personality characteristics under bounded rationality lead to investment mistakes eroding option value. Pessimists tend to underinvest in good opportunities, while optimists overinvest in bad projects. Decisions based on poor information quality can lead to investment error losses that might exceed flexibility value, resulting in *negative* option values. With early exercise, additional timing errors from misestimating continuation value render American options worth less than European. We further examine path dependencies related to prior beliefs, pessimism, narcissism, and myopia.

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## Real Options with Bounded Rationality

### 1. Introduction

Consider the yes or no decision faced by our ancestors millions of years ago to accept an immediate mate or marriage proposal, or that faced by our contemporary CEOs and Boards to accept or reject an exclusive but shortly expiring M&A target proposal. “Yes” (now) or “no” choices are presumably simpler. Imagine how more complicated the decision might get for our ancestors (or even the contemporary CEO) given ambiguous future prospects, imprecise information, cognitive biases and the limited computational or predictive capacity of the human brain if the set of choices expands to “yes” (now), “no”, or “wait” in anticipation of a better mate or M&A target opportunity. The latter decision requires the capability to anticipate and somehow unconsciously quantify the current value of the future prospects. The decision gets even more involved when there are cognitive or psychological biases, e.g., lack of confidence, pessimism or myopia that may increase the chances of the current prospect in the perception of the decision maker. As Herbert Simon (1955) suggested, problems like this can get more manageable if simplified to fit decision-making realities under bounded rationality.<sup>1</sup>

In his path-breaking article in *QJE*, Simon (1955) proposed a behavioral model of rational choice that is compatible with the decision maker’s state of information, limited computational capacity and resulting need for simplification as well as behavioral characteristics of the choosing organization. Simon recognized that some of the constraints taken as givens in an economic optimization problem may be psychological limitations of the organism itself, as they relate to computational and predictive ability or the speed it can learn and adapt. If one delves into the essential elements of rational choice under bounded rationality one might recognize, as Simon did, that what we call the environment (such as informational imprecision and organizational realities) “may lie, in part, within the skin of the organism.” (p. 101).

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<sup>1</sup> For example, the more involved triadic behavioral action problem (“yes”, “no” or “wait”) can be simplified if prospects are seen to arrive sequentially, in which case a “yes” or “no” decision can be made at each decision opportunity. We exploit this feature in our later valuation of the behavioral American option involving invest or wait choices in a backward recursive process that at each decision point reduces to a “0” (no) or “1 (non-zero)” choice. Binary behavior actions for an expiring decision are seen later in the third panel of Figure 1 or the middle panel of Figure 6 for the European options.

Despite the seminal contributions by Herbert Simon (1955) on bounded rationality and his contemporaries Cyert and March (1963) and others on the behavioral theory of the firm more than half a century ago, economics and finance proceeded mostly as usual. Standard economic theory, including Net Present Value (NPV), financial option pricing and Real Options theory (henceforth ROT) widely used in economics, finance and strategy (e.g., Dixit & Pindyck, 1994; Trigeorgis, 1996) have postulated a rational economic decision maker with full information about relevant aspects of his environment operating within well-functioning organizations in perfectly competitive and efficient markets.<sup>2</sup> Assuming traded asset prices fully reflect all relevant information, Black and Scholes and Merton (1973) were subsequently able to price an option through dynamic replication using a portfolio of the underlying asset and riskless borrowing. Essentially the option was priced with reference or relative to the underlying traded asset price, which is observable. Hence the actual growth prospects of the asset are irrelevant for the pricing of the option (and get replaced by the risk-free interest rate) as they are effectively already reflected in the traded asset price. To the extent that futures contracts are also available for certain commodities, the futures price agreed in the market place further provides a direct certainty-equivalent for the uncertain future cash flow prospects, justifying discounting at the riskless interest rate. The insights as well as methodology apparatus of financial options have subsequently been transferred in the context of real options to benefit real asset investment decision-making under uncertainty.

However, there are a number of key limitations in the reality of markets, organizations and human decision making that traditional economics and finance have ignored with occasional catastrophic consequences. First, even in the case of financial options with observable traded asset prices, the observed price may temporarily deviate from the true fundamental value (e.g., when a stock is part of an index subject to computerized portfolio trading), hence the correlation between observed and true prices is not always perfect. Second, even when guided by the most rational and smartest people around (such as the creators of rational option pricing themselves) organizations such as Long Term Capital can go bust in the most liquid of markets even when enlisting the best and most rational managers and computer systems with

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<sup>2</sup> ROT in particular assumes that managers can make better-informed decisions closer to the maturity of their decision horizon (exercise of a real option) and that they act optimally based on reliable observable information in different future economic scenarios –else realized value might be less than theoretical value as the latter is derived assuming optimal exercise decisions.

unprecedented access to information and resources. That underscores that even a small deviation from perfect correlation between the observed and true asset prices or the slightest human error can cost fortunes. Similarly, index futures prices can be disconnected from underlying asset prices due to market moods or interlocking feedback strategies (e.g., of different types of market investors, such as portfolio insurers and arbitrageurs), causing organizations led by world experts in portfolio insurance, such as Leland, O'Brien & Rubinstein, to also get carried by the waves. Third, in a real options context, in many cases a futures contract, the related option and even the underlying asset may not be traded (or even exist, as in R&D). Hence managers must form subjective estimates of the asset value and its future growth prospects based on beliefs, influenced in part by infrequent imperfect observations and prior beliefs and organizational constraints. Fourth, interpretation of events and subjective estimates and beliefs about future asset prospects are subject to cognitive biases and personality flaws. Imperfect correlation between the formed subjective estimates and beliefs and true fundamental asset values can lead to further erroneous decisions, as we elaborate herein. Finally, in a multi-period decision setting, inability to access the “wait-and-see” or future continuation value can lead to additional timing errors and path-dependencies.

In this article we revisit three key aspects of bounded rationality considered by Simon (1955) as representing more realistic environmental and organizational constraints: a) incomplete or imprecise information, a characteristic of the decision maker or of his environment (Simon pp. 100, 102); b) distinguishing between actual and perceived outcomes (p. 102, though Simon stopped short of fully exploring that); and c) “inquiring into the properties of the choosing organism” (p. 100), specifically accounting for cognitive biases (e.g., pessimism, overconfidence or myopia) and irrational personality characteristics (narcissism). Our approach matches closely in spirit and extends Simon’s (1955) simplified and more realistic way of decision making under bounded rationality in the context of real investment decisions under uncertainty.

## **2. Organizational Real Options Reasoning and Belief-based Decision Making**

To set a known benchmark for comparison in our investigation of various organizational realities under imprecise information, recall that in perfect markets and rational decision making incremental asset value adjustments are usually modeled as  $\epsilon$

$\equiv dZ$  (increments of a standard Wiener process), representing random draws from a standard unit normal distribution (e.g., Black and Scholes, 1973; Miller and Arikan, 2004). Future value adjustments are not predictable as they result (only) from unanticipated new signals about asset value and don't depend on past decision maker beliefs or the history of past decisions. In both financial options pricing and in traditional ROT, it is also assumed that the value of the asset over time, and particularly at the times of initial purchase ( $t = 0$ ) and at later option exercise decision times (e.g., at option maturity  $T$ ), is known or can be estimated without bias and that the manager can rely on this unbiased estimate to optimally exercise the option at (if European) or before maturity (if American). There is also an implicit assumption that all asset (project) uncertainty or variability ( $\sigma$ ) is resolvable by the maturity of the option or during the decision-making horizon and that managerial behavior itself would not alter asset (project) uncertainty, which is assumed exogenous to the organization or decision maker.

We next proceed to develop the logic for a Behavior Real Options Theory (BROT) that accommodates the following more realistic assumptions and features about environmental, organizational and human realities. First, we account for partial ignorance in that only part ( $\rho^2$  %) of total asset (project) uncertainty ( $\sigma^2$ ) is explainable or resolvable (i.e.,  $\rho^2\sigma^2$ ) during the decision-making horizon so there is only an imprecise or vague (denoted by  $*$ ) managerial estimate ( $S_t^*$ ) of the true but latent asset value at time  $t$  ( $S_t$ ). The part of total asset uncertainty ( $(1-\rho^2)\sigma^2$ ) that is due to partial ignorance and is not resolvable during the option lifetime is referred to as residual uncertainty or ambiguity. Parameter  $\rho$  ( $0 < \rho < 1$ ) represents the degree of informational (im)precision or resolvable uncertainty, i.e., the proportion of total variation of asset value outcomes resolvable by a management or analyst prediction model and is the correlation between realized asset value outcomes and predicted values (e.g., given by mean analyst forecasts or managerial best-guess estimates). This informational imprecision may reflect environmental and/or organizational realities resulting in an imprecise volatility measurement process under ambiguity. This may be due to other unexplained, unanticipated or unknowable shocks, technological innovations, competitive threats and other exogenous factors external to the firm that are independent from (and orthogonal to) the drivers of the resolvable

asset (project) uncertainty. Hence residual (unknowable) uncertainty is driven by an independent random process,  $\varepsilon_2 \equiv dZ_2$ .

Effectively, superimposing (adding) this residual uncertainty to the resolvable project uncertainty results in an adjusted random process  $\varepsilon (\equiv dZ \equiv) = \rho \varepsilon_1 + (1-\rho^2)^{1/2} \varepsilon_2$ , tying  $\varepsilon_1$  and  $\varepsilon_2$  with correlation  $\rho$ , where  $\varepsilon_1$  and  $\varepsilon_2$  are two independent (uncorrelated) random processes.<sup>3</sup> The higher the correlation between the observed and the true (latent) asset values ( $\rho$ ), the more of the total asset uncertainty is resolvable within the decision horizon and the more confidence the manager has in observing realized future value outcomes and inferring that they are close to the latent true asset values. In a sense  $\rho$  represents the degree of potential learning about the true latent asset value from observing outcome realizations by a rationally adaptive manager. Subjective managerial estimates or perceptions of asset value ( $S_t^*$ ) and the degree of organizational learning and belief adaptation may further be influenced by behavioral biases (e.g., pessimism or optimism) and personality characteristics (e.g., down-to-earth realism and cautiousness, myopia and overconfidence and narcissism) and particularly by bounded (limited) rationality in the form of experiential learning or partial adaptation processes (Levinthal and March, 1981).

In particular, the manager's interpretation, expectation formation and behavioral estimates may be biased by personality characteristics such as ambiguity aversion or gambling-seeking attitudes, overconfidence and narcissism, and clinging to prior beliefs --that might potentially be influenced by past performance such as upward or downward past trends. Hence the perceived expected asset growth and volatility estimates ( $\mu$  and  $\sigma$ ) would no longer remain constant. Moreover, asset (project) growth beliefs and risk characteristics may be affected not only by external asset uncertainty ( $\sigma$ ) factors and managerial flexibility choices (presence of options), but also by (i) the degree of partial ignorance or resolvable uncertainty (proxied by  $\rho$ ) that exogenously constrains the degree of potential organizational learning, (ii) by behavioral biases such as pessimism or optimism (directly affecting the mean growth prospects) and (iii) by ambiguity aversion (cautiousness) or gambling-seeking attitudes that may either scale down or potentially amplify asset (project) uncertainty itself (thereby scaling exogenous asset uncertainty  $\sigma$  by a scalar  $s$  less or greater than

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<sup>3</sup> Alternatively,  $dZ = \rho dZ_1 + \lambda\rho dZ_2$  where  $\lambda \equiv (1-\rho)^{1/2}/\rho$  (such that  $\lambda\rho \equiv (1-\rho)^{1/2}$ ) represents the fraction of residual (unknowable) to resolvable uncertainty, so that the variance of  $dZ$  is  $dt$  and  $Z$  remains a Wiener process.

1). For example, managerial pessimism would erode asset growth expectations reducing future asset value subjective estimates (analogous to a dividend yield,  $\delta^*$ ), so the perceived asset growth drift  $\mu$  would adjust downward to  $\mu^* = \mu - \delta^*$ . This behavioral mean value erosion  $\delta^*$  might be of the form  $-m(\lambda\rho)\sigma$ , where  $m$  ( $-1 < m < 1$ ) is the degree of pessimism (or optimism) biasing the mean (expectations), with the bias rising with the degree of unknowable project uncertainty,  $(\lambda\rho)\sigma$ . For a pessimist (and ambiguity averse) manager,  $m < 0$  and  $\delta^* > 0$  leading to a downward biased subjective assessment of future asset values, whereas for an optimist (e.g., overconfident or narcissist) manager  $m > 0$  and  $\delta^* < 0$  leading to an upward bias in the mean (expectations formation). The degree of cognitive bias in growth expectations formation ( $\delta^*$  or  $\mu^*$ ) is stronger the higher the degree of pessimism or optimism ( $m$ ), the higher the degree of residual (unknowable) uncertainty ( $\lambda\rho$ ), and the higher the total asset (project) uncertainty ( $\sigma$ ).<sup>4</sup>

The degree of organizational learning is thus not only constrained by exogenous environmental or practical forecasting limitations concerning the amount of uncertainty that is resolvable during the decision-making horizon (proxied by  $\rho$ ), but also by managerial and organizational limitations arising from the use of partially adaptive experiential learning processes and incomplete updating of prior-held beliefs. Under bounded rationality a significant bias may arise from organizational learning heterogeneity concerning differences in how slowly decision makers in organizations update their prior beliefs about asset (or option) values in environments characterized by partial ignorance, and from resulting associated decision-making implementation errors in option exercise policies that are based on subjective beliefs as a result of innate vague and inconclusively updated information (driven by  $\rho$ ). If the manager receives updated information from the environment or through specifically-intended learning investments (e.g., a pilot test or due diligence for an M&A target) but the correlation ( $\rho$ ) of the observed outcomes with the true latent value is imperfect and

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<sup>4</sup> Behavioral biases (pessimism or optimism) may also interact with ambiguity aversion (cautiousness) or gambling-seeking attitudes (recklessness) thereby affecting (reducing or amplifying) the second moment (perceived volatility) as well. The impact of ambiguity aversion (measured by  $s$ ), for example, might be linked to the degree of pessimism (proxied by  $m < 0$ ) via  $s = \sqrt{1-m^2}$  (e.g., Kast and Lapiéd (2010)). As the degree of pessimism ( $m$ ) rises, an ambiguity averse and cautious manager would not only be less tolerant of uncertainty (attaching less value to uncertain optional opportunities) but through cautious behavior may endogenously reduce uncertainty from  $\sigma$  to  $s\sigma$ . The reverse might hold in the case of an overconfident and ambiguity-seeking or gambling type manager (leading to  $s > 1$ ). In this case ( $m > 0$ ) for simplicity and symmetry  $s$  might be set to  $\sqrt{1+m^2}$ .

inaccurate ( $\rho$  is low), a reasonable manager may be slow or only partially willing to update prior beliefs. The lower the precision of observed information  $\rho$  the lower the speed of learning adaptation and information updating ( $\alpha$ ) based on new observations by a prudent manager, for whom  $\alpha$  might be correlated with  $\rho$ . Real-life decision makers may also be slow in updating prior beliefs with new information due to risk aversion-driven conservatism, managerial hubris (Herriot and Hambrick, 2005) or organizational inertia due to routine rigidity (Hannan and Freeman, 1977; Nelson and Winter, 1982). Behavioral biases resulting e.g., from pessimism and cautiousness under ambiguity aversion ( $m < 0$ ) that lead to downward biased perceptions ( $E(S_T^*) < S_T$  with  $\delta^* > 0$ ) may be further exacerbated through slower belief updating based on experiential and only partially adaptive learning. This may be more severe the lower the observed information accuracy ( $\rho$ ).

Suppose that managerial beliefs ( $B_t$ ) about asset values after receiving the updated information at time  $t$  follow the partial adjustment process (e.g., see Bush and Mosteller, 1955)  $B_t = \alpha S_t^* + (1-\alpha)B_{t-1}$  (with  $B_0 = S_0^*$  as a base case), where  $S_t^*$  is the observed imprecise asset value estimate (or subjective perception of value) at time  $t$  and  $B_{t-1}$  is the prior-period held organization-formed belief. Alternatively, the updating (change) in beliefs represents only a portion  $\alpha\%$  of the difference (actual incremental learning) between the observed (but likely untrue) value  $S_t^*$  and the prior belief  $B_{t-1}$ , i.e.,  $B_t - B_{t-1} = \alpha(S_t^* - B_{t-1})$  with  $0 < \alpha < 1$ . In a sense, organizationally-formed beliefs follow a process of reverting partially to a moving target of updated new observations—while partly clinging to prior-held beliefs.<sup>5</sup> If all uncertainty is resolvable with the decision time frame and the correlation of observed with true values is perfect ( $\rho = 1$  and  $S_t^* = S_t$ ) an open-minded rational learner would make use of new information updating fully ( $\alpha = 1$ ) such that  $B_t = S_t$ . Under rationality and full updating, a European real call option at maturity ( $T$ ) would be exercised optimally only when  $S_T > K$ , receiving  $NPV_T \equiv S_T - K (> 0)$ . This would result in rationally efficient (optimal) exercise decisions, always investing in good ( $+NPV_T$ ) projects at maturity ( $T$ ) and avoiding bad ( $-NPV_T$ ) ones. But in the presence of residual uncertainty and lack of confidence in the observed value outcomes ( $\rho < 1$ ), the

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<sup>5</sup> In the special case of a rigid or strongly-opinionated manager (e.g., a narcissist) who is reluctant to update their beliefs (or vision of the world) this is equivalent to a mean reverting process to a set (rigid) long-term belief (e.g.,  $B^*$ ).



updating speed based on beliefs might be slower involving only partial adjustment ( $\alpha < 1$ ). This will induce incremental investment errors in the managerial exercise (or termination) of the real option that will realistically (actually) be implemented based on managerial beliefs at the decision horizon (T) when  $B_T > K$ , while still eventually receiving the true project terminal value  $NPV_T = S_T - K$ , that may potentially even be negative. The lower the  $\rho$  the more the manager's inference, interpretation or subjective perception (Cooper and Dunkelberg, 1988) ( $S_t^*$ ) and updated belief ( $B_t$ ) will deviate from the true asset value ( $S_t$ ). If the manager acts on beliefs, therefore, the lower the  $\rho$  the higher the likelihood of making more serious investment errors. At an extreme, if a manager acts on beliefs based on observations that have little correlation to true values ( $\rho = 0$ ), the cost of investment mistakes may dominate flexibility value leading to a *negative* option value as a result of irrational exercise policies.<sup>6</sup>

When the organizational belief at the option maturity ( $B_T$ ) differs from the true but yet unknown asset value ( $S_T$ ) (that will be revealed only subsequently) irrational exercise can result in two types of organizational option exercise decision biases under ambiguity, resulting in over- or under-investment errors: (a) If management exercises the option when it *believes* it is in the money i.e., when  $B_T > K$  but actually  $S_T < K$  (i.e., the option is actually out-of-the-money with  $NPV_T < 0$ ) it will end up overinvesting in a bad ( $-NPV_T$ ) project; and (b) if it allows the option to expire unexercised (terminating the optional opportunity) when it believes  $B_T < K$  but actually  $S_T > K$  (i.e., the option is actually in-the-money or  $NPV_T > 0$ ) it will underinvest missing out on a valuable ( $+NPV_T$ ) opportunity. Either way, the firm will eventually receive the true project value (potentially having  $-NPV_T$  in some states) and will regret these option exercise implementation errors, both of which result in *lower* average realized option value due to informational imprecision and real-life implementation imperfections acting on subjective estimates and beliefs that are vague. The magnitude of the realised investment mistakes and resulting option value erosion can be significant. We estimate the value of these two types of investment mistakes in the simulation-based example in the next section. Both overinvestment and underinvestment bias errors collectively (combined) erode option value from the

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<sup>6</sup> The idea of a negative option value is such an anathema to normative ROT that it led Miller and Shapira (2003) to throw out data on 3 of their 67 behavioral option study respondents who provided negative option values because “option prices should always be non-negative” (p. 275).

complete-information rational exercise decision rule given by standard Black-Scholes option value, i.e.,

$$\text{Actual behavioral option value (C*)} = \text{full-info. rational option value (B\&S) (C)} - (\text{overinvestment mistake} + \text{underinvestment mistake}) \text{ (O+U)} \quad (1)$$

The probability of occurrence and average size of the two mis-investment errors will be affected differentially by different behavioral managerial biases and personality characteristics. Intuitively, an optimist or overconfident manager is more likely to suffer from overinvestment in bad projects (than missing out on good opportunities), whereas an ambiguity averse or pessimist manager to suffer from (over)cautious underinvesting and missing out on good opportunities. The likelihood and average magnitude of both errors will be larger the lower the degree of information precision ( $\rho$ ).

The speed of organizational learning and adjustment of prior beliefs may naturally be lower the lower the observed information accuracy  $\rho$ . Further it may be influenced not only by cognitive biases but by personality characteristics as well, such as in case of strongly-opinionated, one-path vision-driven, overconfident or narcissistic personalities who refuse to give up or are slow to adjust preconceived notions or prior-held beliefs that don't fit their unidirectional mental frame, as well as due to organizational inertia, routine rigidity or other mechanisms leading to overinvestment in the form of escalation of commitment (Staw, 1981; Zardkoohi, 2004). In such situations involving deviations from rationality, the adjustment speed for updating prior beliefs (proxied by  $\alpha$ ) will be slow. Note that although both an overconfident and a narcissist manager may be slow in updating their prior beliefs to (especially unfavorable) developments, the narcissist needs (and responds to) more reinforcement from the environment (Chatterjee and Hambrick, 2007; Aktas et al, 2014) and so past successful performance (or a series of lucky streaks) may reinforce his overconfidence while adverse developments may temper such tendencies.

The rate of updating to new information ( $\alpha_t$ ) may thus be asymmetric, being speedier if developments are favorable (e.g., with  $\alpha_t$  increasing in an upmarket to  $\bar{\alpha}_t$ ) and slower in recognizing and responding to unfavorable developments or adverse market signals ( $\alpha$  reverting to a lower level on down trends). This can lead to

interesting empirical predictions. For example, such asymmetric speed of adjustment would lead to the following testable proposition: narcissist CEOs tend to have a beneficial impact on firm performance in uncertain industry environments with a need to encourage risk-taking and innovative activity, especially when things are going well (on an upmarket). However, their influence may be damaging and occasionally catastrophic when things take a downturn when the CEO is blinded or is too slow to adjust the speed of belief updating. By contrast to the above type of boundedly rational manager committed to a single path or vision of the future (with lower –and potentially asymmetric– adjustment speed  $\alpha$ ) at one extreme, a more rational, down-to-earth or realist type manager who recognizes a more ambiguous and contingent scenario-based or decision-tree like future necessitating more adaptation and learning would be characterized by a higher adaptation rate ( $\bar{\alpha}$ ), resulting in more learning and less mis-investment mistakes and average regret losses.

### 3. Organizational Real Options Modeling

As a benchmark, under perfect markets and rational management the standard true asset value ( $S_t$ ) stochastic process is represented by a random walk or geometric Brownian motion (GBM) of the form:

$$dS/S = \mu dt + \sigma dW \quad (2)$$

where  $dW$  is an increment of a standard Wiener process, and in discrete time by

$$\ln S_T - \ln S_0 = \mu \Delta t + \sigma \sqrt{\Delta t} \varepsilon \sim N[\mu \Delta t, \sigma \sqrt{\Delta t}] \text{ or } S_T = S_0 \exp(\mu \Delta t + \sigma \sqrt{\Delta t} \varepsilon) \quad (3)$$

where  $\mu \equiv r - \frac{1}{2}\sigma^2$  under risk-neutrality characterizing efficient markets and rational managers (with  $\varepsilon$  representing random draws from a standard normal distribution).

This process leads to the theoretical Black-Scholes value for a European call option on a non-dividend paying asset (for efficient markets and rational managers),

$$C_0 = S_0 N(d_1) - Ke^{-r\tau} N(d_2) \quad (4)$$

where  $d_1 = \{\ln(S_0/K) + (r + \frac{1}{2}\sigma^2)\tau\} / \sigma\sqrt{\tau}$ ;  $d_2 = d_1 - \sigma\sqrt{\tau} = \{\ln(S_0/K) + \mu\tau\} / \sigma\sqrt{\tau}$

with  $N(d_2)$  being the probability that the option will be exercised in a rationally efficient (risk-neutral) world where the realized asset value at the option exercise is known with no ambiguity.<sup>7</sup>

In a behavioral ROT based on the more realistic organizational assumptions discussed in the previous section, the stochastic process for the observed (not true) asset value estimate or subjective perception of value ( $S_t^*$ ) is now adjusted as follows:

$$dS^*/S^* = \mu^* dt + s\sigma dZ, \quad \text{with } dZ = \rho dZ_1 + \lambda\rho dZ_2 \quad (2')$$

where  $\mu^* \equiv \mu - \delta^*$ ,  $\delta^* \equiv \delta^*(m, \rho, \sigma)$  is the degree of behavioral bias,<sup>8</sup>  $m$  ( $-1 < m < 1$ ) the degree of managerial pessimism ( $m < 0$ ) or optimism ( $m > 0$ ),  $s$  the volatility-altering impact of ambiguity-driven conservatism or gambling-seeking attitudes, and  $\rho$  the correlation (degree of information accuracy or potential learning) between the realized value outcome ( $S_t^*$ ) and the true latent (unknowable) value ( $S_t$ ).<sup>9</sup> This results in the following behaviorally-adjusted process generating future observations or realizations of asset values:

$$S_T^* = S_0^* \exp(\mu^* \Delta t + s\sigma\sqrt{\Delta t} \varepsilon) \quad (3')$$

$$\text{with } \varepsilon = \rho \varepsilon_1 + \lambda\rho \varepsilon_2 \quad \text{with } \lambda\rho = (1 - \rho^2)^{1/2} \quad (5)$$

and  $\varepsilon_1$  and  $\varepsilon_2$  being independent random draws (linked together in  $\varepsilon$  with correlation  $\rho$ ). For a conservative or cautious manager volatility-scalar  $s < 1$ , while for overconfident, narcissist and risk-seeking managers  $s > 1$ .<sup>10</sup>

<sup>7</sup> In the actual risk averse world  $r$  is replaced by the actual asset growth rate.

<sup>8</sup> An intuitive choice is  $\delta^* = -m(\lambda\rho)\sigma$ .

<sup>9</sup> Suppose that in Eq. (2) under ambiguity and assuming all uncertainty is resolved by option maturity (see Kast et al, 2014; Driouchi, Trigeorgis and Gao, 2014)  $dW = m dt + \rho dZ_1$ . Then Eq. (2) under ambiguity becomes:  $dS/S = \mu dt + \sigma (m dt + \rho dZ_1) = (\mu + m\sigma) dt + (\rho\sigma) dZ_1$ . Next, additionally assuming that only a part of uncertainty is resolvable (knowable) during the option horizon ( $\rho\sigma$ ) while the rest ( $\lambda\rho\sigma$  or  $(1-\rho)^{1/2}\sigma$ ) is residual (non-resolvable) uncertainty and is additively independent (in line with and extending Posen, Leiblein and Chen, 2014, Eq. (8) with  $n = \lambda\rho\sigma$ ), we add term  $(\lambda\rho)\sigma dZ_2$  to the previous equation, giving the following stochastic process for the true (but unknown) asset value ( $S$ ):  $dS/S = (\mu - \delta) dt + \sigma dZ$ , where  $\delta = -m\sigma$  and  $dZ = \rho dZ_1 + \lambda\rho dZ_2$ . The corresponding process for the observable but imprecise asset value outcome ( $S^*$ ) when only a part ( $\rho\sigma$ ) rather than all volatility ( $\sigma$ ) is resolvable, is then obtained by replacing  $\sigma$  with  $\rho\sigma^*$  in above. Ambiguity cautiousness or gambling-seeking attitudes can scale  $\sigma$  to  $\sigma^* = s\sigma$ .  $\delta^*$  can be obtained as  $-m n$  or  $-m\lambda\rho\sigma$ .

<sup>10</sup> There is abundant evidence that overconfident and narcissistic managers may be prone to increased risk taking and thus raising volatility ( $s > 1$ ). Overconfident CEOs take on more debt (Malmendier and Tate, 2010) and undertake more risky projects (Hirschleifer et al, 2010), with their risk taking enhanced

The resulting actual behavioral option value ( $C_0^*$ ) is always lower than the rational (optimal) Black-Scholes value of Eq. (4), partly as a result of the two types of investment mistakes that erode option value under partial ignorance, behavioral biases and limited rationality.<sup>11</sup> Besides the standard option inputs ( $S_0$ ,  $K$ ,  $r$ ,  $\tau$ ,  $\sigma$ ), the behavioral probability of option exercise based on beliefs and the resulting behavioral option value now also depend on the degree of informational imprecision ( $\rho$ ) as well as on the behavioral biases of the manager such as pessimism vs. optimism or overconfidence (via  $m$  and  $\delta^*$ ). Heterogeneous organizational option values thereby result from a nonlinear merging of standard option parameters with the above behavioral and organizational factors under ambiguity. As a result, decision makers in real organizations (as well as firms and investors in imperfect factor markets) will deviate from standard rationality assumptions under complete information and value an organizational real option *below* the theoretical rationally-efficient (risk-neutral) price obtained by the Black-Scholes model of Eq. (4). The degree of behavioral option value erosion depends on the degree of imprecision in managerial ability to estimate resolvable uncertainty within the decision horizon, managerial ambiguity conservatism or gambling-seeking attitudes and behavioral biases and personality characteristics such as overconfidence or narcissism.

As noted, interesting and non-standard predictions obtain from a behavioral ROT.<sup>12</sup> As the degree of information imprecision rises ( $\rho$  declines), the likelihood and cost of investment errors increases and so behavioral option value ( $C^*$ ) tends to decline ( $\partial C^*/\partial \rho > 0$ ). Investment mistakes are more severe when behavioral biases (pessimist or optimism) are stronger as they cause deviations from true rational value realization. The effect ( $\partial C^*/\partial m$ ) can be quite different (reverse) for overconfident (and especially ambiguity-seeking CEOs such as narcissists) compared to ambiguity-averse pessimist managers. Pessimist and cautious managers are more likely to miss out on

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by standard compensation contracts (Gervais et al, 2011). Narcissist CEOs have a propensity to take on more risks (Li and tang, 2010) and the operating performance of firms led by narcissist CEOs is more extreme and more volatile.

<sup>11</sup> The perceived ( $C^*$ ) value of a real-life behaviorally-biased real option adjusted for the above assumptions and behavioral biases under ambiguity aversion and partial ignorance (with resolvable uncertainty only the fraction  $\rho\sigma$ ) is of the form:  $C_0^* = S_0^* e^{-\delta^*\tau} N(d_1^*) - Ke^{-r\tau} N(d_2^*)$  where  $d_1^* = \{\ln(S_0^*/K) + (r - \delta^* + \frac{1}{2}\sigma^{*2})\tau\}/(\rho\sigma^*)\sqrt{\tau}$ ;  $d_2^* = d_1^* - (\rho\sigma^*)\sqrt{\tau} = \{\ln(S_0^*/K) + \mu^*\tau\}/(\rho\sigma^*)\sqrt{\tau}$  with  $\sigma^* = s\sigma$ ,  $\mu^* = \mu - \delta^*$ ,  $\delta^* \equiv \delta^*(m, \rho, \sigma)$ ,  $\mu \equiv r - \frac{1}{2}\sigma^2$ .  $N(d_2^*)$  is the behavioral probability that the option will be exercised based on managerial beliefs (i.e.,  $B_T > K$ ) assuming full updating ( $\alpha = 1$  or  $B_T = S_T^*$ ) and no influence of ambiguity-driven attitudes on asset volatility ( $s = 1$ ).

<sup>12</sup> These can be obtained by examining the partial derivatives of the analytical expression in footnote 11, such as  $\partial C/\partial \rho$  and  $\partial C/\partial m$ , and are verified by our simulation results discussed in the next section.

potentially lucrative investment opportunities than overinvesting. The cost of underinvestment will generally dominate overinvestment under risk aversion. Overconfident managers, on the other hand, will tend to perceive higher option value and are more likely to commit overinvestment mistakes than underinvesting ones. These mistakes can potentially even lead to negative option values if observed signals have little correlation with true asset values.

For practical valuation purposes and to illustrate the value impact of some of the previously discussed behavioral investment biases, such as overinvestment in bad projects or underinvestment in good opportunities, and associated path dependencies related to updating of beliefs under bounded rationality and related asymmetric mechanisms (e.g., higher speed of updating following good developments or slow adjustment on the downside by a narcissist), we illustrate next valuation of a simple European call option using properly-adjusted Monte Carlo simulation –first in the standard benchmark case without, and then with, inclusion of various organizational realities, behavioral characteristics, and limited rationality belief-dependent exercise biases, path dependencies and associated mis-investment mistakes. We subsequently present an extension for the American option. Certain analogies between our problem setup and Simon’s (1955) bounded rationality framework are noteworthy. Simon’s possible future states of affairs is represented by the actual ( $S_t$ ) or perceived ( $S_t^*$ ) random value outcomes (or managerial beliefs  $B_t$  about them) implemented via stochastic simulation paths (see later Figures 1, 6 and 7). The set of behavior alternatives or actions in the real options context is the binary choice to do something (invest or exercise an option, getting some non-zero immediate or cash value) now or not (getting no immediate cash value or 0) at the maturity of an expiring or European option; or potentially to choose later (getting a wait-and-see option or future continuation value) in the American option context. In the spirit of Simon (1955), the latter problem is simplified in a recursive manner (see Appendix).

### ***Simulation Model***

We employ simulation as a simple, yet powerfully illustrative methodology well accepted in economics and finance (e.g., Longstaff and Schwartz, 2001) as well as in management research (e.g., Miller and Arikan, 2004; Sakhartov and Folta, 2014). In what follows we use Monte Carlo simulation for valuing a behavioral European real call option (later extended to the American case) by simulating random paths for the

observed asset value  $S_T^*$  at the exercise time (maturity)  $T$  based on Eq. (3') in such a way that it has correlation  $\rho$  with the true terminal asset value  $S_T$  of Eq. (3). This is implemented as described below (see Figure 1), with actual numbers referring to the row of path 1. The following base-case parameters are assumed:  $S_0 = 100$ ,  $K = 110$  (so  $NPV_0 = S_0 - K = -10$ ),  $T = \Delta t = 1$ ,  $\sigma = 0.30$ ,  $r = 0.02$ ,  $\rho = 0.9$  (and  $\lambda = 0.48$ ),  $m = -0.5$  (pessimist, with  $s = 0.87$ ),  $\delta^* = 0.07$ ,  $\alpha = 0.9$ . Two sets (columns) of random draws from a standard normal distribution are created in Excel using command `=NORM.INV(RAND(); 0;1)`, in row of path 1 shown as  $\varepsilon_1 (\Delta Z_1) = 0.838$  and  $\varepsilon_2 (\Delta Z_2) = -0.158$  in columns B and C. Then a revised  $\varepsilon^* (\Delta Z^*) = 0.685$  is obtained from Eq. (5) in column D with  $\rho = 0.9$ . Columns E and F estimate the noise terms for the true asset value process ( $S_t$ ) based on  $\sigma\sqrt{\Delta t} \varepsilon_1 (= 0.30\sqrt{1} \cdot 0.838 = 0.251)$  and the observed behavioral process ( $S_t^*$ ) based on  $s\sigma\sqrt{\Delta t} \varepsilon^* (= 0.87 \times 0.30\sqrt{1} \cdot (0.685) = 0.178)$ , linked with correlation  $\rho = 0.9$ . Columns G and H add the expected growth (drift) terms of the true and behavioral processes ( $\mu = (r - \frac{1}{2}\sigma^2)\Delta t = -0.025$  and  $\mu^* = (r - \delta^* - \frac{1}{2}s^2\sigma^2)\Delta t = -0.079$ ) to their respective noise terms ( $\Delta S = -0.025 + 0.251 = 0.226$ ,  $\Delta S^* = -0.079 + 0.178 = 0.099$ ). Then columns I and J take exponents to these powers giving the terminal true and (correlated) observed or perceived asset values  $S_T (= \exp(0.226)) = 125.392$  and  $S_T^* (= \exp(0.099)) = 110.389$ . Column K estimates organizational belief based on  $B_T = \alpha S_T^* + (1-\alpha)B_0 (= 0.9 \times 110.389 + 0.1 \times 100) = 109.350$  assuming  $B_0 = S_0^* = S_0 = 100$  and  $\alpha = 0.9$ . Column L gives the full information exercise payoff,  $\max(S_T - K, 0) = \max(125.392 - 110) = 15.392$ , approximating the Black-Scholes value. Column N gives as a benchmark for comparison the perceived option value payoff (had the manager received the perceived value upon exercise, which is not true) as  $\max(B_T - K, 0) = \max(109.350 - 110, 0) = 0$ . Column N gives the actual or realized behavioral option payoff acting on the belief  $B_T (= 109.350)$  of column K (rather than the true value  $S_T = 125.392$  of column I), while actually receiving the true project value  $S_T - K$  if invest. Actual exercise, however, is based on the decision rule if  $B_T > K$  invest and get  $S_T - K$  (even if negative), else don't invest and get 0. Here this gives 0 since  $B_T = 109.350 < K = 110$ .

Column O estimates the *overinvestment* (O) error based on the rule if  $B_T > K$  &  $S_T - K < 0$  get  $-NPV_T = S_T - K$ , else invest properly and get 0 error. Here this results in 0 overinvestment since  $B_T = 109.350 < 110$ . Column P gives the *underinvestment* (U) error based on the rule if  $B_T < K$  &  $S_T - K > 0$  don't invest and lose  $+NPV_T (= S_T -$

K), else invest properly and get 0. Here this results in missing out on a good opportunity with  $+NPV_T = 125.392 - 110 = 15.392$ . Column Q gives their sum or total (mis-investment) error ( $O + U$ ),  $0 + 15.392 = 15.392$ . A similar process is repeated for paths 2-4 with no investment mistakes. However, path 5 now results in an overinvestment mistake of 20.016 since belief  $BT = 110.174 > K (=110)$  leading to erroneous overinvesting actually resulting in  $NPV_T = S_T - K = 89.984 - 110 = 20.016$ . The above process is repeated for 20,000 sets of random draws (paths), then taking the average of the full information (B&S) and the actual behavioral option values (and the investment errors) and discounting at the riskless interest rate ( $r = 0.02$ ) back to the current time ( $t = 0$ ) to obtain the current worth of the European option values and the associated investment errors.

#### 4. Basic Valuation Results and Robustness

We next present basic valuation results for the European call option. As shown in Figure 2 Panel A (for a pessimist with  $m = -0.5$ ), the resulting current value of the behavioral option (averaged across 20,000 paths)  $C^*$  is 7.27 (also % of the asset value of  $S_0 = 100$ ), which is about 18% below the estimated full information Black-Scholes benchmark of 8.86 (%) shown as the top dotted line. The current cost of the overinvestment error (O) seen on the lowest curve in Panel A is 0.16 (2%), and of the underinvesting mistake (U) is 1.45 (16%). As seen in Panel B, there is a higher probability (13%) for this pessimist manager (with  $m = -0.5$ ) of committing an underinvestment (U) mistake than an overinvestment (O) one (2%). Panel C shows that the average magnitude of a typical underinvestment error is only marginally higher (12.02 vs. 10.63) --and that this may reverse for an extreme optimist. The sum of both over- and under-investment mistakes ( $O + U$ ) is 1.61 (18% of B-S). As Figure 2 confirms, when the manager is unbiased ( $m = 0$ ), the two errors are of roughly equal probability (about 7%) and of low impact (less than 0.50 or 5%), achieving the highest realized organizational option value ( $C^*$ ) and narrowing the shortfall below the Black-Scholes value (C). By contrast, for an optimist manager ( $m = +0.5$ ) facing the same exact option with identical terms the O error dominates the U mistake both in terms of likelihood and value impact, confirming a reversal as hypothesized. It is worth noting that in Figure 2 panel C, the size of the two investment errors is roughly the same across the broad range of behavioral biases ( $m$ ) and that it increases steeply



for extreme risk-aversion (pessimism). Option value ( $C^*$ ) is highest for an unbiased manager which faces least investment mistakes.

Figure 3 Panel A confirms (for the above pessimist manager with  $m = -0.5$  and  $\rho = 0.9$ ) that the average size and value of both investment errors increase with exogenous asset (project) uncertainty ( $\sigma$ ). Moreover, Panel B shows that while the probability of O error rises with asset uncertainty, the probability of U error declines, raising the possibility of a non-monotonic relation of  $C^*$  with  $\sigma$  in some circumstances, in contrast to standard ROT.<sup>13</sup> That is, exogenous uncertainty  $\sigma$  may increase both the flexibility value and the mis-investment losses in a potentially non-monotonic way. Figure 4 shows that even in the case of an unbiased manager ( $m = 0$ ), while  $C^*$  is increasingly falling short of the rational Black-Scholes solution ( $C$ ) as the degree of informational precision ( $\rho$ ) declines,  $C^*$  declines into negative territory at low levels of exogenous asset volatility ( $\sigma$ ). The combination of high informational imprecision (very low  $\rho$ ) leading to large investment mistakes and low uncertainty leading to low flexibility value can drive  $C^*$  into negative territory even for an unbiased manager. Behavioral biases (pessimism or optimism) may increase the likelihood and cost of investment mistakes, thus increasing the likelihood of encountering *negative* option values.

Figure 5 shows what happens to the above underinvestment (U) and overinvestment errors (O) (Panel A) and their probabilities (panel B) for the above pessimist manager (with  $m = -0.5$ ) as the quality of information deteriorates from  $\rho = 0.9$  (in above base case) to lower degrees of informational precision. As hypothesized, when informational precision declines from  $\rho 0.9$  down to  $0.1$ , the likelihood of both types of investment errors increases dramatically (from 13% to 28% for U and from 2% to 8% for O from Panel B), with their value (cost) impact (in Panel A) also rising commensurably from 1.45 (16% of Black-Scholes value) to 7.1 (80% of B-S) for U and from 0.16 (2%) to 2.14 (24% of B-S) for O error. Total investment error losses (T) rise from 1.61 (18%) to 9.24 (104% of B-S) at  $\rho = 0.1$ . For extreme informational imprecision (essentially forming and acting on beliefs based on junk information) at  $\rho < 0.15$ , total investment error (T) exceeds the full flexibility value of the Black-Scholes call option, throwing the realized organizational option value (declining solid

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<sup>13</sup> Regarding the relation between uncertainty, option value and investment, see also Folta (1998), Folta and O'Brien (2004) and Lee and Makhija (2009).

dark line)  $C^*$  into *negative* territory, in line with our hypothesis on the logical plausibility of *negative* option values under ambiguity.

The case of an overconfident CEO (with  $m = +0.5$ ) facing an otherwise identical acquisition option as above is particularly instructive showcasing an anatomy of an overinvestment crime leading to negative option values. We assume this overconfident manager has a moderately high response speed to new information ( $\alpha = 0.7$ ) despite extreme informational imprecision ( $\rho = 0.1$ ). His optimistic bias leads him to arrive at a perceived call option value of  $C^P = 12.22$ , almost a 40% premium above the Black-Scholes call value of  $C_0 = 8.86$ . His 30% probability of overinvestment results in an O damage alone of 7.25, alone wiping out over 80% of the Black-Scholes flexibility value. With an additional 4.0 value loss from underinvestment (with a 16% probability), the total error of 11.26 by far surpasses the 8.86 flexibility value, resulting in a negative option value of -2.4. So that the reader is not inclined to attribute this negative option calculation (resulting as the discounted average of 20,000 random simulation paths) due to some technical modeling error, the logic behind effected scenarios leading to such huge potential misinvestment losses, when acting on beliefs based on low quality information, can be traced in some of the highlighted simulation paths (in rows 7, 11 and 22) shown in Figure 6.

Path (row) 7 shows an unlikely (but possible) scenario with the belief  $B_T = 90.311$  being below the exercise cost ( $K = 110$ ), leading to an erroneous decision not to invest. But with a true high asset value ( $S_T = 161.491$ ), this results in a huge underinvestment loss (from a missed good opportunity) of 51.491 (column P). The actual payoff received acting on beliefs is 0 (column N), when having full (reliable) information would have resulted in an  $NPV_T$  payoff of 51.491 (column L). Rows 11 and 22 show more likely scenarios of exaggerated beliefs (in both  $B_T > K = 110$ ) leading to erroneous overinvestment by this overconfident manager. In Path 11 the true asset value is quite low ( $S_T = 86.378$ ), resulting in  $-NPV_T = S_T - K = 86.378 - 110 = -23.622$ , another heavy loss again resulting from a wrong exercise decision (albeit of the opposite type). In path 22 the manager finally gets the exercise decision right (since  $B_T$  as well as  $S_T$  are both above 110), but he overestimates the payoff to be 30.821 (column M), while the true (and actual) payoff is 23.243 (columns L and N). There is no investment (option exercise) error in this case, but the wrong perception of value (based on erroneous beliefs) leads to an exaggerated perceived option value of  $C_0^P = 12.22$  (when the 20,000 path values of column M are averaged

and discounted), exceeding the full information option value (based on the average discounted paths of column L with  $\rho = 1$ ) of 8.86, as also confirmed by the Black-Scholes formula. When the actual payoffs from acting on beliefs but receiving the true project values  $NPV_T$  (positive or negative) of column N are averaged across all paths and discounted, the negative behavioral option value of -2.4 results. This is not a calculation mistake, but the result of tracking decision mistakes under information imprecision, irrationality and behavioral biases. Investment mistakes get exacerbated and additional timing errors arise when there are earlier decision opportunities due to human error and bias in assessing the future value of continuing vs. the NPV of investing now in the case of the American option.

### ***Additional Biases from Early Exercise: American Behavioral Option***

When the call option can be exercised at multiple earlier times prior to maturity  $T$  (i.e., it is American rather than European call), then the exercise decision (invest or wait) is applied at each earlier period  $t$  ( $< T$ ). When a rational manager has full information at the time of earlier exercise ( $t$ ), it is now not sufficient to invest (exercise) if  $S_t > K$  or  $NPV_t > 0$ . Rather, it should do so only if  $NPV_t$  additionally exceeds the expected continuation (call option) value conditional on the asset value at  $t$  ( $S_t$ ),  $E(C_t | S_t)$ . Even when  $NPV_t > 0$  but the discounted expected call option value from continuing to next period ( $C_{t+1} e^{-r\Delta t}$ ) is higher, a rational manager with flexibility and foresight under uncertainty would still choose to wait, receiving 0 cash flow in the current period.<sup>14</sup> Setting a 0 for the option cash flow (or  $NPV_t$ ) at the early decision time  $t$  indicates *no* investment at this time (though investment may be made later on). Under perfect information when the manager knows the true asset value ( $S_t$ ), the conditional expectation  $E(C_t | S_t)$  can be estimated from a cross-sectional OLS regression of  $C_t$  ( $= C_{t+1} e^{-r\Delta t}$ ) values against  $S_t$  across all simulation paths as in Longstaff and Schwartz (2001). The above foresight-type estimation presents an extra behavioral complication and opens up room for further exercise decision and optimal timing errors when the expected continuation value estimate,  $E(C_t) (\equiv E(C_t | S_t))$ , is subject to error and cognitive bias. These errors will further be exacerbated when the manager acts (exercises the American-type option) based on imprecise observations or subjective estimates ( $S_t^*$ ) and acts on subjective beliefs that suffer from behavioral

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<sup>14</sup> Of course if  $S_t < K$  (or  $NPV_t < 0$ ) the manager would not exercise early (getting 0 payoff in the current period  $t$ ) and would rather choose to wait (at  $t = T$  would terminate the project).

biases ( $B_t$ ). We consider two variants of estimating expected continuation (option) value: (i) based on observations of asset values ( $S_t^*$ ) and the resulting option estimates ( $C_t^*$ ), namely  $E(C_t^* | S_t^*)$ , and (ii) based on organizational beliefs of asset value ( $B_t$ ) and associated expected option beliefs  $E(C_t^B) \equiv E(C_t^B | B_t)$ . In Figure 7 Panel B we present results based on the latter.

The revised early exercise decision rule at time  $t$  ( $< T$ ) for the behavioral American option based on beliefs is:

If  $B_t > K$  (i.e.,  $NPV_t^B > 0$ ) and  $NPV_t^B > E(C_t^B | B_t)$  get  $NPV_t = S_t - K$ ; else get 0 now.

That is, if perceived NPV based on beliefs ( $NPV_t^B \equiv B_t - K$ ) at earlier decision time  $t$  is positive and exceeds the expected belief-based option value from continuing,  $E(C_t^B)$ , exercise now or invest early (getting the actual or true NPV eventually), else wait and receive 0 value now. This replaces the earlier simpler decision rule for the European option (difference in italics): If  $B_t > K$  get  $NPV_t = S_t - K$ ; else get 0 (at  $t = T$ ). Note that the above covers the subcases

If  $B_t < K$  (i.e., perceived  $NPV_t^B \equiv B_t - K < 0$ ) do not exercise (hence receive 0) now;  
 If  $B_t > K$  (i.e., perceived  $NPV_t^B > 0$ ) but  $NPV_t^B < E(C_t^B | B_t)$  wait (receive 0 now).

The revised behavioral *Underinvestment* (U) mistake when acting on beliefs leads to not investing and missing out on a good project, is now based on the rule: if  $B_t < K$  or  $NPV_t^B (\equiv B_t - K) < E(C_t^B)$  &  $NPV_t (\equiv S_t - K) > E(C_t) (> 0)$  (rather than 0 as in the European option case) don't exercise (get 0) now and lose  $-NPV_t (\equiv S_t - K)$ , else invest properly and get 0 error. The underinvestment error always leads to value destruction (loss of good projects). The revised behavioral *Overinvestment* (O) error i.e., investing based on beliefs when should't have, is now based on the rule: if  $NPV_t^B (\equiv B_t - K) > E(C_t^B) (> 0)$  &  $NPV_t (\equiv S_t - K) < E(C_t)$  (rather than  $< 0$  as in European) then get (true)  $NPV_t (\equiv S_t - K)$ , else (invest properly and) get 0 error. Note that resulting  $NPV_t$  from an overinvestment mistake can be either damaging if  $S_t < K$  or it can be beneficial partly due to lack in case  $S_t$  (being  $< K + E(C_t) > K$ ).

The American option is valued following a backward recursive process described in more detail in the Appendix.<sup>15</sup> Figure 7 Panels A-C illustrate this process

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<sup>15</sup> Reading the Appendix carefully and accompanying illustrative Figure 7 at this point is essential for the reader who wants to delve and confirm the details of the valuation process. The detailed valuation process was moved to the Appendix so as not to interrupt the flow and focus on differential results

for valuing an (otherwise identical one-year) American call option with 2 steps (involving semi-annual managerial decisions) (referred to herein as A2) allowing potential early exercise at intermediate decision point  $t = 1$ . Panel D illustrates valuation of an equivalent American option with 4 decision steps (involving quarterly managerial decision reviews), referred to as A4.

The left two main sections of Panel C in Figure 7 show the resulting time-0 cash flows or cumulative discounted forward-NPV's across the 2 decision points for the first 10 simulation paths, first based on full information and then when acting on beliefs. These resulting cash flows for the 2-step American option (A2) also indicate the optimal decision rule (non-zero if invest or 0 if not exercise) at each respective decision time (1 or 2) as well as the value created (or lost) from making that decision at each time: 0 indicates do *not* invest at this time, while a non-zero (positive or negative) value indicates the value created or lost from that specific investment decision. Comparisons and differences among the decisions based on full information compared to acting on biased beliefs can be easily tracked in Panel C.

There are a number of noteworthy situations leading to investment errors shown in Figure 7 Panel C. For example:

**Costly underinvestment error.** Path (row) 1 shows that the firm should have invested at  $t=2$  (receiving 8.46) but, acting on beliefs, did not invest at all resulting in costly 8.46 U error (Column L). The manager does not invest based on beliefs at  $t=2$  since  $NPV_2^B = 109.02 - 110 = -0.98 < 0$  (shown in Column I in Panel A), when he should since true  $NPV_2 = S_2 - K = 118.63 - 110 = 8.63$  at  $t = 2$  or 8.46 discounted at  $t = 0$ .

**Costly overinvestment error.** Path (row) 3 shows the firm should not have invested at any time but the manager, acting on beliefs, erroneously invested at  $t=2$ , suffering an overinvestment loss of 23.87 (Column I). Here the manager invests at maturity based on his beliefs ( $NPV_2^B = 113.25 - 110 = 3.25 > 0$ ), when he should *not* have invested since true  $NPV_2 = S_2 - K = 85.65 - 110 = -24.45$  at  $t = 2$ , leading to an overinvestment loss of 23.87 when discounted at  $t = 0$ .

**Lucky overinvestment error.** Path (row) 5 shows that the firm should not have invested at all but the manager, acting on beliefs, invested erroneously at  $t = 1$ , benefiting by an NPV of 10.24, a negative error resulting by luck (shown in Column

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given in next section. For the reader interested primarily in the comparative results and resulting managerial implications the detailed Appendix might be skimmed or skipped.

H). The manager invests erroneously based on his beliefs since  $NPV_1^B = 139.91 - 110 = 29.91 (> 0)$ . However, he should *not* have invested at  $t = 1$  since true  $NPV_1 = S_1 - K = 120.35 - 110 = 10.35$  (Column C in Panel B), which is less than full-information continuation value,  $E(C_1) = 22.13$ . By luck the firm benefits by the positive  $NPV_1$  of 10.35 from investing at  $t = 1$ , which discounted to  $t = 0$  gives a lucky overinvestment error of 10.24.

**Costly delayed timing error.** Path (row ) 6 shows the firm should have invested at time 1 (getting an NPV of 27.08) but due to erroneous beliefs invests with delay at time 2, resulting in a 2.96 O loss at that time. The 27.08 U error from missing out on a good opportunity at  $t=1$  plus the 2.96 loss from the erroneous investment decision at  $t =2$  result in a total misinvestment error  $T = 30.04$  (Column N).

**Lucky early timing error.** Path (row) shows that the firm should have optimally exercised at maturity  $t =2$  but instead erroneously invested early at  $t =1$ . A lucky 24.16 O gain from the premature investment at  $t=1$  (Column H) more than offsets the 15.46 opportunity loss of not investing at time 2 (Column L), resulting in a net (total) lucky gain (negative error) of  $T = -8.69$  (Column N).

Similar results for the American call option with 4 (quarterly) rather than 2 (semi-annual) managerial decision points (A4) are shown in Panel D. Paths with interesting decision differences are again highlighted. For example, path 2 shows lucky early investment timing (at  $t =2$  instead of 3); path 4 shows a 1.18 U error from never investing (when should have invested at maturity  $T = 4$ ); paths 5 and 6 show lucky O timing errors at maturity (when should have never invested); path 8 shows a costly net 37.84 O error from investing a year earlier than optimally ( $t=1$  instead of 2); path 10 shows a beneficial (negative) 5.73 O error from investing at  $t=1$  when should never have invested.

The average values across the 20000 paths of the full information and behavioral American options (A2 and A4) and the associated U, O and Total investment mistakes for the given set of parameters are given at the bottom of Panels C (for A2) and D (A4), respectively. Adjusting for early exercise allows examining interesting comparative implications associated with early exercise biases and variations in key value drivers, such as information precision ( $\rho$ ) and cognitive biases ( $m$ ). It also allows examining path dependent and asymmetric dynamic effects related to initial beliefs ( $B_0$ ) or asymmetric belief adjustment (varying  $\alpha$ ) depending on upside or downside past trends and strong- opinionated personality characteristics as

in case of a narcissist CEO, and managerial myopia from underweighting the continuation value.

Figure 8 confirms and illustrates our above conjecture violating another sacred basic principle of standard option pricing, that an American call option allowing for more (earlier) exercise decision choices can be worth less (rather than more) than its European counterpart due to additional timing and human comparison errors in making the now vs. later choice, even when informational precision is very high but not perfect ( $\rho = 0.9$ ). Panel A shows that the value of the American option with 4 decision opportunities (A4) is below that with 2 decision choices (A2), which tracks the European option value close but from below, with the deviations (losses from decision mistakes) being greater for an optimist (higher  $m$ ). The next three panels (B, C and D) show the value impact, probability and average size of errors for the American behavioral option with 2 steps (A2) for different degrees of pessimism or optimism ( $m$ ). Although these are broadly comparable in shape to the corresponding panels (A, B and C) for the European counterpart of Figure 2, one difference is noteworthy. Panel B, which tracks the value of errors, shows that although a pessimist suffers more from U errors, these tend to be partly offset by beneficial (negative) O errors that might be attributed to luck. These can be traced to the average size (Panel D) rather than the probability of errors (panel C) –the latter being of similar shape as in the European option case.

Figure 9 Panel A again confirms, in the case of the pessimist (for  $m = -0.5$ ), that A4 remains below A2 and E as option values decline with deteriorating levels of informational precision ( $\rho$ ), with values turning negative when  $\rho$  drops below 0.2. Panel B shows that option values and errors for A2 are analogous to Figure 5 (for the European option), except for beneficial overinvestment errors arising at high levels of  $\rho$  for the pessimist.

Figure 10 examines dynamic path dependencies related to initial beliefs ( $B_0$ ) for a pessimist ( $m = -0.5$ ), an optimist ( $m = +0.5$ ) and a neutral manager ( $m = 0$ ) when information precision is high ( $\rho = 0.7$ ). Panel A shows that when initial beliefs are unbiased ( $B_0 = S_0 = 100$ ), as assumed in our base-case analysis, the European call option value for an optimist (O) or a Pessimist (P) are close (just below) that of a neutral manager. However, as initial beliefs ( $B_0$ ) get lower (than 100) for the pessimist or higher for the optimist, European option value gets eroded as the current belief at maturity ( $B_T$ ) is more biased (being a weighted average of the initial biased

belief  $B_0$  and current observation  $S_t^*$ ). Panel B shows that in the case of the American option with 4 belief revision opportunities this value erosion due to initial belief bias is mitigated as a result of the dynamic adjustment of beliefs. For example, a pessimist with more pessimistic (lower) initial beliefs will have a lower probability of overinvestment error for the American option with 4 revision opportunities.

Figure 11 Panel A examines how European (E) and American (A2) behavioral option values vary with organizational learning or adjustment rate policy ( $\alpha$ ) under bounded rationality for an overconfident manager ( $m = +0.5$ ) when there is good information quality ( $\rho = 0.7$ ). For the European option (E) there is an optimal adjustment rate ( $\alpha^*$ ) around 0.4, but for the American option with 4 revision opportunities the optimal rate is lower. Panel B shows that for the European option total error (T) is lowest around 0.4, where the probabilities of O and U errors are about equal (Panel C).

## 5. Further Discussion and Implications

Let us revisit the case of an overconfident CEO (with  $m = +0.5$ ) facing a low degree of informational precision ( $\rho = 0.1$ ) but using a high rigid adjustment rate ( $\alpha = 0.7$ ). Had the overconfident CEO's organization been more conservative in responding to the new (but highly unreliable) observations (e.g., acting more on consensus building among the top executive team), using a low response rate to match the degree of informational imprecision (e.g.,  $\alpha = \rho = 0.1$  rather than  $\alpha = 0.7$ ), the probability and cost of overinvestment mistakes for the European option would decline from 30% to 2.5% and from 7.25 to 0.58, respectively. As the investment errors decline, the realized option value  $C^*$  increases by 2.34. But if the CEO is not just overconfident (biased upward in his expectations) but also a narcissist that suppresses different opinions (Park, Westphal and Stern, 2011) and insists on a high responsiveness to the new (but unreliable) information ( $\alpha = 0.7$ ), then the incremental impact of his narcissist personality above the overconfidence tendency would lead to an incremental overinvestment loss of 6.67 and a further reduction in option value of 2.34. If the call option was related to a target acquisition (and the unit was in \$100s of million), then the extra feature of narcissism over mere overconfidence would have cost the firm \$2.34 million in target opportunity mispricing.

Another interesting question is, how much should the organization be willing to pay for a learning capability-type investment that would improve the informational



precision from  $\rho = 0.1$  to  $0.7$  (given its narcissist CEO insists on an adaptation rate of e.g.,  $\alpha = 0.5$ )? Alternatively, how much should a firm pay for due diligence raising  $\rho$  from  $0.1$  to  $0.7$  before deciding to exercise an option on an acquisition target? The extra option value ( $\Delta C^*$ ) created from improving the learning precision from  $\rho = 0.1$  to  $0.7$  (for a narcissist CEO with  $m = +0.5$  assuming a fixed adaptation rate  $\alpha = 0.5$ ) is  $7.20 (=5.29+1.91)$  (in \$100s of million) for this European option, so the organization should invest in this learning capability if feasible provided the cost of doing so is less than  $7.20$ . The value increase is a bit higher for an American option as more decision mistakes can be avoided through improved information accuracy.

$\rho$	European (C*)	A (2-steps)	A (4-steps)
0.1	-1.91	-2.38	-2.94
0.7	5.29	4.91	4.40
$\Delta$	7.20	7.29	7.34

We next contrast the asymmetric adjustment to new information in the case of an organization lead by a narcissist CEO compared to an otherwise identical overconfident --but not narcissist-- CEO (with  $m = +0.5$ ) assuming good informational precision ( $\rho = 0.7$ ). Specifically, we illustrate the effect on the realized option value for the case of an American option with 2 periods (A2). We assume that the narcissist CEO's ego, clinging on beliefs (with  $B_0 = 100$  and  $K = 110$ ) and suppression of opposing opinions (Park, Westphal and Stern, 2011) are such that they are boosted following good recent past performance (2 up moves in  $B_t$ ) thereby resulting in a low rate ( $\alpha = 0.1$ ) of adjusting to new observations; a moderate organizational adjustment rate ( $\alpha = 0.5$ ) following roughly level past performance (1 up and 1 down move); and the CEO taking a more quiet and back-seat role allowing a greater organizational willingness to learn from new information ( $\alpha = 0.9$ ) in case of sustained poor past performance (2 downward moves). Compared to an otherwise identical overconfident ( $m = +0.5$ ) but not narcissist CEO pursuing a moderate fixed adjustment policy of  $\alpha = 0.5$  (median), the loss from U mistakes for the narcissist CEO is higher ( $6.62$  vs.  $3.63$ ) and behavioral option value (A2) is lower ( $3.37$  vs.  $4.91$ ).<sup>16</sup> The additional characteristic of narcissism above mere overconfidence bias in this case would cost the firm an option value loss of  $1.54$ . Even though the overconfident and narcissist CEOs share many attributes (e.g., both may be inclined

<sup>16</sup> In the down scenarios the belief  $B_T$  tends to be below  $K$  leading to higher underinvestment errors particularly in cases when true  $S_T$  far exceeds cost  $K$ .

to exaggerate expectations and be slow in updating their beliefs to unfavorable developments), as the narcissist needs more reinforcement from the environment (Chatterjee and Hambrick, 2007; Aktas et al, 2014) past successful performance (or a series of lucky streaks) may worsen his overconfidence and adaptation bias here.

Figure 12 examines how behavioral option value ( $C^*$ ) varies with the degree of managerial myopia in the case of an organization lead by an overconfident CEO with a fixed moderate adjustment rate ( $\alpha = 0.5$ ) in case of high informational precision ( $\rho = 0.9$ ). Managerial myopia here leads to underweighting the expected belief-based continuation value ( $bE(C_t^B)$ , with a myopia coefficient weight  $b < 1$ ). When  $b = 1$  (no myopia),  $A_4$  and  $A_2$  are close to  $E$ , as obtained previously. But when managerial myopia is severe ( $b$  gets closer to 0) leading to a lower weight on future continuation value in making the early exercise decision at time  $t$  based on  $NPV_t^B$  vs.  $bE(C_t^B)$ , belief-based exercise or continuation decisions are more prone to errors and option value gets further eroded. The more decision revision times subject to severe managerial myopia during the decision horizon, the lower the behavioral option value ( $A_4 < A_2 < E$ ).

Similar investment mistakes as discussed above (possibly exacerbated due to the strategic context or leveraging) may also result in the acquisition (as well as the exercise) of options, such as in patents or target companies as in the above M&A context, as the actual or true option value under ambiguity conditions  $C_0^*$  (barring synergies) would be less than the theoretical full-information rational (Black-Scholes) option value  $C_0$ , even if (as in the case of overconfidence) perceived option value  $C_0^P$  may exceed  $C_0$ . In such cases, even paying conservatively the Black-Scholes option price ( $C_0$ ) –even if much less than the perceived value (by an overconfident CEO)  $C_0^P$ – may actually be an overpayment. Behavioral heterogeneity by potential bidders for acquiring a strategic option (e.g., an M&A acquisition) can result in overpayment, contributing to the winner’s curse.<sup>17</sup> Paying the rational Black-Scholes price ( $C_0$ ) to an ambiguity averse seller for an acquisition may be paying too much (since actual  $C_0^* < C_0$ ), but doing so by an overconfident or narcissist CEO may be perceived as a bargain (since  $C_0^P > C_0$ ), conveying a false hope that it may lead to a competitive advantage in strategic resource markets.

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<sup>17</sup> If a manager is overconfident ( $m > 0$  and  $\delta^* < 0$ ), the expected growth perception will be higher ( $\mu^* > \mu$ ) and the asset value at maturity will be overestimated on average ( $E(S_T^*) > S_T$ ), leading to potential overestimation of perceived option value ( $C_0^P > C_0$ ).

## 6. Conclusions

We have taken up a main research challenge in economics (Simon, 1995), finance (Trigeorgis, 1996) and strategy (e.g., Bowman and Moscovitz, 2001; Miller and Arikian, 2004; Miller and Shapira 2004; Adner and Levinthal, 2004; McGrath et al, 2004) calling for research examining pragmatic investment decisions in real organizations. We posited that managerial decisions to acquire (dispose) and exercise real options are based on organizational beliefs. Beliefs are updated based on prior experience and new information, but the effectiveness of organizational belief updating and of the exercise decision depend on the precision of the new information. The less precise the quality of information, the more imprecise the beliefs and the bigger the investment errors based on these beliefs. We have additionally accounted for the impact of cognitive or behavioral limitations and personality characteristics that may further bias (color) managerial beliefs downward or upward, such as due to conservatism or overconfidence, potentially even influencing endogenously (altering the scale of) risk exposure itself and the speed of belief updating.

We have shown that under ambiguity surrounding a real investment decision based on biased managerial beliefs the impact of uncertainty on behaviorally-driven option values is moderated by the degree of information imprecision, behavioral investor or personality characteristics and biases and associated indirect effects related to ambiguity cautiousness or risk-taking attitudes driven by bounded rationality. In line with Simon (1955), our behavioral real option valuation (or BROT) represents more realistically the type of organizational decision rules and heuristics that might be followed by human decision makers in real organizations characterized by partial ignorance, bounded rationality and behavioral biases.

Our findings contribute to the literature in several ways. We explicitly model and examine the role of informational imprecision ( $\rho$ ) in driving investment errors resulting from forming and acting on beliefs based on imperfect observations or subjective estimates and tracing the interactions of informational imprecision with behavioral biases, personality characteristics and adaptive learning. We show how informational imprecision (low  $\rho$ ) makes belief updating ineffective and exacerbates these investment mistakes, eroding behavioral option value  $C^*$ . At an extreme (at very low  $\rho$ ), we demonstrate the logical plausibility that European behavioral option value ( $C^*$ ) may get *negative* as the investment mistakes might dominate any flexibility

value left if the quality of information is bad. Moreover, we demonstrate that the American option can be worth less than the European due to timing and human errors in comparing future continuation value to current NPV. These are non-conventional revelations. We developed and illustrated a simple simulation methodology to determine the \$value of the investment mistakes and quantify the option value realized ( $C^*$ ). For the European call case this confirms the Black-Scholes solution when our organizational assumptions concerning information imprecision ( $\rho = 1$ ), behavioral bias ( $m = 0$ ) and adaptive learning ( $\alpha = 1$ ) are relaxed. For the American option, we extended the Longstaff and Schwartz (2001) simulation methodology to account for information imprecision, behavioral and personality biases in early exercise decisions focusing in human errors in assessing expected continuation value of the wait-and-see option. These extensions allowed us to look deeper into the path dependent and asymmetric dynamic effects related to initial beliefs, asymmetric belief adjustment depending on upside or downside past trends and strong-opinionated personalities such as narcissist CEOs. We also had a glimpse into managerial myopia from underweighting the continuation value.

We limited our scope herein to the analysis of a single call option situation in isolation, to focus and uncover the interacting effects of informational imprecision, belief updating, cognitive biases and personality characteristics in the simplest terms. Our core analysis was thus limited by a number of simplifying assumptions, which simultaneously present opportunities for future research. Specifically, our analysis can be extended in several directions. First, BROT opens up new research territory regarding the management debate on the boundaries of the applicability of ROT (e.g., Adner and Levinthal, 2004; McGrath, Ferrier and Mendelow, 2004) and the concurrent examination of rational, behavioral and organizational explanations of decision making (Elfenstein and Knott, 2014). Second, asset values and beliefs (updating expectations) might be modeled following different processes e.g., incorporating higher moments (such as higher skewness from effective exercise of prior options) or mean reversion to long-term held beliefs (potentially also incorporating discontinuities from technological or other shocks). Third, BROT might be extended and integrated with learning (e.g., Bayesian) type investments that allow obtaining more precise estimates of asset (project) value, at a cost. This might shed more light on the value of learning-type investments like due diligence prior to an acquisition and give a more positive spin on firm heterogeneity and competitive

advantage under informational imprecision and cognitive biases. Fourth, our approach can be extended further to the dynamic analysis of multi-stage option investments to address in more depth behavioral phenomena such as short-termism (Levinthal and March, 1993; Laverty, 1996; Miller, 2002; Malmendier and Tate, 2010) or escalation of commitment (Staw, 1981; Zardkoohi, 2004; Sinha, Inkson and Barker, 2014). Fifth, it may be used to revisit strategy formulation and execution through the options lens seen as behavioral sequential switch options (Bowman and Hurry, 1993; Trigeorgis, 1996). Sixth, there is related need to examine decision biases as they affect interacting firm portfolio options such as to defer, grow, shut down and switch (Trigeorgis, 1993; Oriani and Sobrero, 2008). Seventh, BROT can be further merged with game theory and IO principles to examine behavioral option games, focusing on biases, investment mistakes and further distortions in decision (exercise) rules in the presence of competitive rivalry or collaborative behavior. Eighth, future research might examine theoretical and empirical analogues in the financial options setting, including due diligence decisions in M&As or executive stock options and market option transactions in own shares by top executives (Jagolinzer, Larker and Taylor, 2011; Cohen, Malloy and Pomorski, 2012). Finally, our BROT predictions can be further examined in behavioral lab experiments, managerial survey work, and via large empirical studies accounting for behavioral and personality characteristics, particularly focused on top executives and their effect on firm performance (Chatterjee and Hambrick, 2007; Mackey, 2008). Such efforts should enhance our understanding of how organizations make decisions and how they can more effectively acquire and exercise real options for enhanced competitive advantage.

## Appendix: Valuation of Behavioral American Option by Simulation

This Appendix presents an extension of the Longstaff and Schwartz (2001) simulation methodology to valuing the behavioral American option that allows quantifying investment mistakes arising from behavioral biases and timing errors under imperfect information. The discussion is focused around the American equivalent of an otherwise identical call option as in the earlier example (with  $K = 110$ ,  $T = 1$  year,  $B_0 = 110$ ) involving an optimist manager ( $m = +0.5$ ). Other parameters are the same.

Figure 7 Panel A shows the stochastic time (path) evolution of the true asset value ( $S_t$ ), the subjective estimate or imperfect observation ( $S_t^*$ ) and the managerial belief ( $B_t$ ) over the two decision time periods at intermediate time  $t=1$  and at maturity ( $T=2$ ) for A2. The panel shows  $S_t$  (True),  $S_t^*$  (Observed) and  $B_t$  (Belief) at  $t = 1$  and 2 for the first 10 paths out of 20000 randomly generated paths for an optimist manager ( $m = +0.5$ ) under high information precision  $\rho = 0.9$  and high learning rate  $\alpha = 0.7$ . For example, for path (row) 1,  $S_1 = 92.20$  and  $S_2 = 118.63$ , while  $B_1 = 92.40$  and  $B_2 = 109.02$ . For convenience, Column D gives the NPV of investing at maturity ( $T=2$ ), i.e.,  $NPV_2 = S_2 - K = 118.63 - 110 = 8.63$  (discounted at  $t=1$  this is 8.55 and at  $t = 0$  it is 8.46). Column I gives the  $NPV_2^B$  based on beliefs at  $T = 2$ ,  $NPV_2^B = B_2 - K = 109.02 - 110 = -0.98$ . These values are used in deriving the option value or cash flows at earlier periods. Under full information (and no bias), the payoff of the European option at the end of each path at maturity ( $T=2$ ) is simply  $\text{Max}(NPV_2, 0)$ ; when averaged across the 20000 paths and discounted to  $t=0$  this approximates the Black-Scholes value (8.86). For the behavioral case acting on beliefs (with  $m = +0.5$ ) it is based on  $\text{Max}(NPV_2^B, 0)$  instead and results in value 7.38 (summarized in Figure 7 panel A).

Panel B of Figure 7 examines the decision to exercise early or continue (wait-and-see) at the intermediate decision period ( $t = 1$ ), first under full information (left panel) and when there is behavioral bias acting on beliefs (right panel). This decision involves a key comparison between the  $NPV_1$  of investing immediately (at  $t=1$ ) and the future or expected continuation option value from waiting and deciding next period based on better information,  $E(C_1)$ . Estimating the latter introduces additional human error when acting on beliefs, exacerbating the investment mistakes due to informational imprecision and timing errors. Under full information and rational decision making, this conditional expectation can be estimated from a least-squares regression of (discounted next-period) realized option values ( $C_{t+1}$ ) (Column D) against realized asset values ( $S_t$  in Column B or  $S_t^*$  in Column E of Panel A)

analogous to Longstaff and Schwartz (2001). For example, in path (row) 2,  $C_1 = C_2 \exp(-r \Delta t) = 30.87 \exp(-0.02 \times 0.5) = 30.56$  (with  $\Delta t = T/N = 1/2$ ). A short-cut can be achieved by noting that when  $NPV_1 (= S_1 - K)$  is negative or when  $S_1 < K$ , as in path (row) 1 (also 3, 4 and 9) the manager should not exercise early (at  $t = 1$ ) under full information and the option cash flow at  $t = 1$  or  $NPV_1$  from immediate exercise is thus set to 0. Expected continuation value,  $E(C_1)$ , is also set to 0 in this case.<sup>18</sup> Thus, for simplification in the spirit of Simon (1955), entries for  $S_t$ ,  $NPV_t$ ,  $C_{t+1}$  and  $E(C_t)$  in the left part of Panel B are set to 0 when  $NPV_1 < 0$ . Following Longstaff and Schwartz (1981), with full information on asset values  $S_t$ , the conditional expectation function  $E(C_t) \equiv E(C_t | S_t)$  is estimated through a least-squares regression of discounted next-period realized option values ( $C_{t+1}$ ) along (conditional on) each stock path ( $S_t$ ). The result of a simple OLS regression estimation (shown at bottom left in Panel B) is:  $E(C_t | S_t) = -0.78 + 0.216 B_t$ . The resulting expected continuation value for each path under full information is shown in Column E in panel B.

A similar process is used for obtaining the behavioral American option value acting on beliefs, except that the criterion now is instead  $NPV_1^B (= B_1 - K) < 0$ . That is, when  $B_1 < K$ , as in path (row) 1 (also 3 and 4) of the right part of Panel B, the manager will not exercise early (with potential bias or error). Again as a shortcut, the entries for  $B_t$ ,  $NPV_t^B$ ,  $C_{t+1}^B$  and  $E(C_t^B)$  for paths 1, 3 and 4 are set to 0. Given bounded rationality, the expected continuation value based on beliefs ( $B_t$ ) can at best be estimated by (implicitly) inferring a linear pattern between future projections of belief-based option values ( $C_{t+1}^B$ ) and current beliefs ( $B_t$ ) i.e., by subconsciously mimicking an OLS regression linear fit among columns H (with  $C_{t+1}^B$  as y variable) and F (with  $B_t$  as x variable). The result is  $E(C_t^B | B_t) = -0.534 + 0.188 S_t$  (bottom right in Panel B). Expected continuation value based on beliefs for each path is shown in Column I in Panel B. Then for each path, the optimal decision at the early intermediate decision time ( $t = 1$ ) under full information is to select the best (noted by a \*) of either exercising immediately to obtain  $NPV_1$  or waiting and receiving  $E(C_1)$ . Similarly, when acting on beliefs ( $B_t$ ), the optimal decision is  $\max(NPV_1^B, E(C_t^B))$ .

Note that for paths 1, 3 and 4 in Panel B the decision *not* to invest (with a 0 in column C for  $NPV_1$  or 0 in Column G for  $NPV_1^B$ ) is the same; there is no

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<sup>18</sup> The decision choices and valuation results are generally robust to whether these negative  $NPV_1$  paths are set to 0 and are included in the regression to estimate  $E(C_t)$ , left out of the regression as missing observations or the actual values for  $S_t$  and  $C_{t+1}$  are used so we here present the short-cut variant (also in line with Longstaff and Schwartz, 2001).

opportunity cost in these cases since the true  $NPV_1 (= S_1 - K)$  is 0. In general, the manager will *not* invest (at  $t = 1$ ) based on beliefs (hence receiving a 0 in Column G) if  $NPV_1^B < E(C_1^B)$ . For path 9 the decisions may potentially differ. With full information when  $S_1 = 105.06 (< K=110)$  or with true  $NPV_1 = 105.06 - 110 = -4.94$  the firm should not invest (getting instead  $NPV_1 = 0$ ), whereas with erroneous belief of 128.67 ( $> 110$ ) and  $NPV_1^B = 18.67 (> 0)$ , the firm might potentially invest (if it does so, however, it would receive true  $NPV_1 = -4.94$ ). To decide whether to invest or wait, the firm will compare belief  $NPV_1^B$  (18.67) with belief-based expected continuation value,  $E(C_1^B)$ , estimated at 27.01 (Column I). Since  $NPV_1^B < E(C_1^B)$  as  $18.67 < 27.01$ , the firm would choose not to invest at  $t = 1$ . In panel C at row (path) 9 this is indicated as receiving an immediate cash flow ( $NPV_1$ ) of 0 (Column E in Panel C row 9). Thus in the end the exercise decision at  $t=1$  for path 9 turns out identical (i.e., *0* or *no* invest) when acting on beliefs (Column E in Panel C) as when under full information (Column B). No mis-investment mistake therefore results whatsoever (albeit aided by some luck) at  $t=1$ . For path (row) 9 there is an overinvestment error at  $t=2$ , however, of 15.33. That is because at maturity  $T = 2$ ,  $B_2 = 129.37$  hence belief-based  $NPV_2^B = B_2 - K = 19.37 (> 0)$  so the firm will invest, though it turns out in a bad project since true  $NPV_2 = S_2 - K = 94.36 - 110 = -15.64$ ; discounted back for 2 periods, this behavioral decision at  $T=2$  results in a time-0 value loss of 15.33 (Column F in Panel C row 9). This is also recorded as an overinvestment error of 15.33 in Column I (panel B row 9).

The sum of the time-0 discounted option cash flows or discounted NPVs at decision times 1 and 2 are given in Column D of Panel C for full information (0 for row 9) and in Column G for behavioral (-15.33). The behavioral option value in each path is the full information value minus the sum of the Overinvestment (O) and the Underinvestment (U) errors cumulated over the (here 2) decision periods. The same procedure is applied for the other paths (the first 10 paths are shown for verification in Figure 7). When averaged across the 20000 paths, panel C column D gives the full information American call option value (8.59), which approximates the Black-Scholes formula in absence of “dividend” effects; Column G gives the behavioral American call option value with 2 steps (A2) of 7.32; column J gives the Overinvestment error (-0.39), Column M the underinvestment error (1.66), and Column N the Total error (1.27). Again, eq. (1) applies, namely  $7.32 = 8.59 - 1.27$ .



## REFERENCES

- Adner R, Levinthal DA. 2004. What is not a real option: considering boundaries for the application of real options to business strategy. *Academy of Management Review* 29(1): 74–85.
- Agliardi E, Sereno L. 2011. The effects of environmental taxes and quotas on the optimal timing of emission reductions under Choquet-Brownian uncertainty. *Economic Modelling* 28: 2793-2802.
- Aktas N, de Bodt E, Bollaert H, Roll R. 2014. CEO narcissism and the takeover process. *Journal of Financial and Quantitative Analysis* (forthcoming).
- Barnett M.L. 2008. An attention-based view of real options reasoning. *Academy of Management Review*, 33: 606–628.
- Barney J. 1986. Strategic factor markets: expectations, luck, and business strategy. *Management Science* 32: 1231–1241.
- Barney JB. 1991. Firm resources and sustained competitive advantage. *Journal of Management* 17(1): 99–120.
- Black F, Scholes M. 1973. The pricing of options and corporate liabilities. *Journal of Political Economy* 81(3): 637–654.
- Bowman EH, Hurry D. 1993. Strategy through the option lens. *Academy of Management Review* 18(4): 760–782.
- Bowman EH, Moskowitz GT. 2001. Real options analysis and strategic decision making. *Organization Science* 12(6): 772–777.
- Busby JS, Pitts CGC. 1997. Real options in practice. *Management Accounting Research* 8: 169–186.
- Bush RR, Mosteller F. 1955. *Stochastic Models for Learning*. John Wiley & Sons.
- Camerer C, Lovallo D. 1999. Overconfidence and excess entry. *American Economic Review* 89(1), 306–318.
- Chatterjee A, Hambrick DC. 2007. It's all about me: narcissistic CEOs and their effects on company strategy and performance. *Administrative Science Quarterly* 52 (3): 351–386.
- Chi TL, McGuire DJ. 1996. Collaborative ventures and value of learning. *Journal of International Business Studies* 27(2): 285–307.
- Chi TL. 2000. Option to acquire or divest a joint venture. *Strategic Management Journal* 21(6): 665–687.
- Coff RW, Laverty KJ. 2007. Real options meet organizational theory. *Advances in Strategic Management* 24: 333–361.

- Cohen L, Malloy C, Pomorski L. 2012. Decoding Inside Information. *Journal of Finance* 67(3): 1009–1043.
- Cooper AC, Dunkelberg WC, Woo CY. 1988. Entrepreneurs' Perceived Chances for Success. *Journal of Business Venturing* 3(2): 97–108.
- Cuypers IRP, Martin X. 2010. What makes and what does not make a real option? *Journal of International Business Studies* 41(1): 47–69.
- Cyert RM, March JG. 1963. *A Behavioral Theory of the Firm*. Englewood Cliffs, NJ: Prentice-Hall.
- Davis JP, Eisenhardt KM, Bingham CB. 2009. Optimal structure, market dynamism, and the strategy of simple rules. *Administrative Science Quarterly* 54: 413–452.
- Dierickx I, Cool K. 1989. Asset stock accumulation and sustainability of competitive advantage. *Management Science* 35: 1504–1511.
- Dixit AK, Pindyck RS. 1994. *Investment under Uncertainty*. Princeton, NJ: Princeton University Press.
- Driouchi T, Trigeorgis L, Gao Y. 2014. Choquet-based European option pricing with stochastic (and fixed) strikes, OR Spectrum.
- Elfenbein D, Knott A-M. Time to exit: rational, behavioral, and organizational delays. *Strategic Management Journal*, forthcoming.
- Folta TB. 1998. Governance and uncertainty: the trade-off between administrative control and commitment. *Strategic Management Journal* 19: 1007–1028.
- Folta TB, O'Brien JP. 2004. Entry in the presence of dueling options. *Strategic Management Journal* 25: 121–138.
- Folta TB, Miller KD. 2002. Real options in equity partnerships. *Strategic Management Journal* 23(1): 77–88.
- Gervais S, Heaton JB, Odean T. 2011. Overconfidence, compensation contracts, and capital budgeting. *Journal of Finance* 66(5): 1735–1777.
- Graham JR, Harvey CR. 2001. The theory and practice of corporate finance: evidence from the field. *Journal of Financial Economics* 60: 187–243.
- Herriot SR, Levinthal D, March JG. 1985. Learning from experience in organizations. *American Economic Review* 75(2), 298–302.
- Hiller NJ, Hambrick DC. 2005. Conceptualizing executive hubris. *Strategic Management Journal* 26: 297–319.
- Hirshleifer D, Low A, Teoh SH. 2010. Are overconfident CEOs better innovators? Working Paper.

- Howell SD, Jagle AJ. 1997. Laboratory evidence on how managers intuitively value real growth options. *Journal of Business Finance and Accounting* 24: 915–935.
- Jagolinzer AD, Larker DF, Taylor DJ. 2011. Corporate Governance and the Information Content of Insider Trades. *Journal of Accounting Research* 49 (5): 1249-1274.
- Kahneman D, Slovic P, Tversky A (eds). 1982. *Judgment under Uncertainty: Heuristics and Biases*. Cambridge University Press: Cambridge, U.K.
- Kahneman D, Tversky A. 1979. Prospect theory: an analysis of decision under risk. *Econometrica* 47(2): 263–291.
- Kast R, Lapied A, Roubaud D. 2014. Modelling under ambiguity with dynamically consistent Choquet random walks and Choquet-Brownian motions. *Economic Modelling* 38: 495-503.
- Kets de Vries MFR, Miller D. 1985. Narcissism and leadership. *Human Relations* 38(6), 583–601.
- Kogut B. 1991. Joint ventures and the option to expand and acquire. *Management Science* 37(1): 19–33.
- Kogut B, Kulatilaka N. 2001. Capabilities as real options. *Organization Science* 12: 744–758.
- Laverty KJ. 1996. Economic ‘short-termism’. *Academy of Management Review* 21: 825–860.
- Lee S-H, Makhija M. 2009. The effect of domestic uncertainty on the real options value of international investments. *Journal of International Business Studies* 40: 405–420.
- Leiblein MJ, Miller DJ. 2003. An empirical examination of transaction and firm-level influences on the vertical boundaries of the firm. *Strategic Management Journal* 24(9): 839–859.
- Levinthal DA, March JG. 1981. A model of adaptive organizational search. *Journal of Economic Behavior and Organization* 2: 307–333.
- Levinthal D, March JG. 1993. The myopia of learning. *Strategic Management Journal* 14(2): 95–112.
- Li J, Tang Y. 2010. CEO hubris and firm risk taking in China: the moderating role of managerial discretion. *Academy of Management Journal* 53(1), 45–68.
- Lippman S, Rumelt R. 1982. Uncertain imitability: an analysis of interfirm differences in efficiency under competition. *Bell Journal of Economics* 13(2): 418–453.
- Longstaff F, Schwartz E. 2001. Valuing American options by simulation: a simple Least-Squares approach. *Review of Financial Studies* 14(1):113–147
- Mackey A. 2008. The effect of CEOs on firm performance. *Strategic Management Journal* 29: 1357–1367.

- Malmendier U, Tate G. 2005. CEO overconfidence and corporate investment. *Journal of Finance* 60(6), 2661–2700.
- Malmendier U, Tate G, Yan J. 2010. Managerial beliefs and corporate financial policies. *NBER Working paper* 13124.
- March, J. 1996. Learning to be risk averse. *Psychological Review* 103(2): 309–319.
- March JG., Olsen JP. 1975. The uncertainty of the past: organizational learning under ambiguity. *European Journal of Political Research* 3(2): 147–171.
- March JG, Shapira Z. 1987. Managerial perspectives on risk and risk taking. *Management Science* 33: 1404–1418.
- March J., Simon H. 1958. *Organizations*. New York: Wiley.
- McGrath RG. 1997. A real options logic for initiating technology positioning investments. *Academy of Management Review* 22(4): 974–996.
- McGrath RG, Nerkar A. 2004. Real options reasoning and a new look at the R&D investment strategies of pharmaceutical firms. *Strategic Management Journal* 25: 1–21.
- McGrath R, Ferrier W, Mendelow A. 2004. Real options as engines of choice and heterogeneity. *Academy of Management Review* 29(1), 86–101.
- Merton RC. 1973. Theory of rational option pricing. *Bell Journal of Economics and Management Science* 41: 141-183.
- Miller KD. 2002. Knowledge inventories and managerial myopia. *Strategic Management Journal* 23(8): 689–706.
- Miller KD, Folta TB. 2002. Option value and entry timing. *Strategic Management Journal* 23(7), 655–665.
- Miller KD, Arikan AT. 2004. Technology search investments: evolutionary, option reasoning and option pricing approaches. *Strategic Management Journal* 25: 473–485.
- Miller KD, Shapira Z. 2004. An empirical test of heuristics and biases affecting real option valuation. *Strategic Management Journal* 25(3): 269–284.
- Morf C, Rhodewalt F. 2001. Unraveling the paradoxes of narcissism. *Psychological Inquiry* 12(4): 177–196.
- Nickerson J., Zenger TR. 2004. A knowledge-based theory of the firm: the problem-solving perspective. *Organization Science* 15(6): 617–632.
- Nelson R, Winter S. 1982. *An Evolutionary Theory of Economic Change*. Harvard University Press: Cambridge, MA.
- Oriani R, Sobrero M. 2008. Uncertainty and the market valuation of R&D within a real options logic. *Strategic Management Journal*, 29: 343–361.

- Park SH, Westphal JD, Stern I. 2011. Set up for a fall: the insidious effects of flattery and opinion conformity toward corporate leaders. *Administrative Science Quarterly* 56 (2): 257–302.
- Peteraf MA. 1993. The cornerstones of competitive advantage: a resource-based view. *Strategic Management Journal* 14: 179–191.
- Posen HE, Leiblein MJ, Chen JS. 2014. A behavioral theory of real options. Working Paper.
- Reuer JJ, Tong TW. 2005. Real options in international joint ventures. *Journal of Management* 31(3): 403–423.
- Reuer JJ, Tong TW. 2007. How do real options matter? Empirical research on strategic investments and firm performance. *Advances in Strategic Management* 24: 145–173.
- Reuer JJ, Leiblein MJ. 2000. Downside risk implications of multinationality and international joint-ventures. *Academy of Management Journal* 43(2), 203-214.
- Rumelt RP, Schendel DE, Teece DJ, eds. 1994. *Fundamental Issues in Strategy*. Boston, MA: Harvard Business School Press.
- Sakhartov A, Folta T. 2014. Resource relatedness, redeployability, and firm value. *Strategic Management Journal* 35: 1781-1797.
- Scherpereel C. 2008. The option-creating institution: a real options perspective on economic organisation. *Strategic Management Journal* 29: 455–470.
- Simon H. 1955. A behavioral model of rational choice. *Quarterly Journal of Economics* 69: 99–118.
- Sinha PN, Inkson K, Barker, J. 2014. Committed to a failing strategy. *Organization Studies* 33(2): 223–245.
- Smit H.T.J., Trigeorgis L. 2004. *Strategic Investment: Real Options and Games*. Princeton, NJ: Princeton University Press.
- Staw BM. 1981. The escalation of commitment to a course of action. *Academy of Management Review* 6: 577–587.
- Tetlock P. 2000. Cognitive biases and organizational correctives. *Administrative Science Quarterly* 45: 293–329.
- Triantis A. 2005. Realizing the potential of real options: does theory meet practice? *Journal of Applied Corporate Finance* 17: 8–16.
- Triantis A, Borison A. 2001. Real options: state of the practice. *Journal of Applied Corporate Finance* 14(2): 8–24.
- Trigeorgis, L. 1993. The nature of option interactions and the valuation of investments with multiple real options. *Journal of Financial and Quantitative Analysis* 28

(1):1-20.

Trigeorgis L. 1996. *Real Options: Managerial Flexibility and Strategy in Resource Allocation*. Cambridge, MA: MIT Press.

Tversky A, Kahneman D. 1974. Judgment under uncertainty: heuristics and biases. *Science* 185: 1124–1130.

Vassolo RS, Anand J, Folta TB. 2004. Non-additivity in portfolios of exploration activities: a real options-based analysis of equity alliances in biotechnology. *Strategic Management Journal* 25: 1045–1061.

Wernerfelt B. 1984. A resource-based view of the firm. *Strategic Management Journal* 5: 171–180.

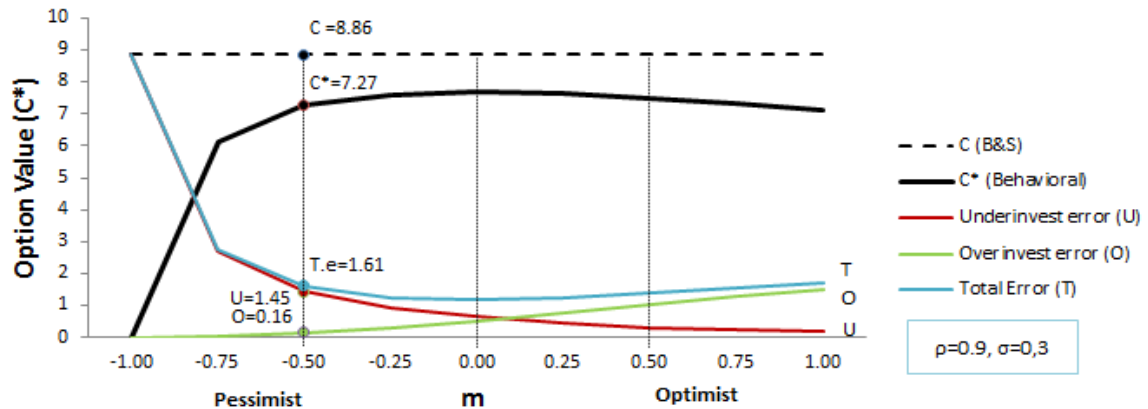
Zardkoohi A. 2004. Do real options lead to escalation of commitment? *Academy of Management Review* 29(1): 111–119.

Figure 1. Random path simulation of asset values at European call option maturity (T) (true value  $S_T$  (true), correlated observed or subjective estimate  $S_T^*$ ), beliefs ( $B_T$ ) and resulting option values (Black & Scholes full info. payoff C, perceived option value  $C^P$  and actual behavioral option value acting on beliefs  $C^*$ ) along with resulting Overinvestment (O), Underinvestment (U) and Total misinvestment errors (T) for a pessimist manager ( $m = -0.5$ ) with high information precision ( $\rho = 0.9$ ) and high learning rate ( $\alpha = 0.9$ ).

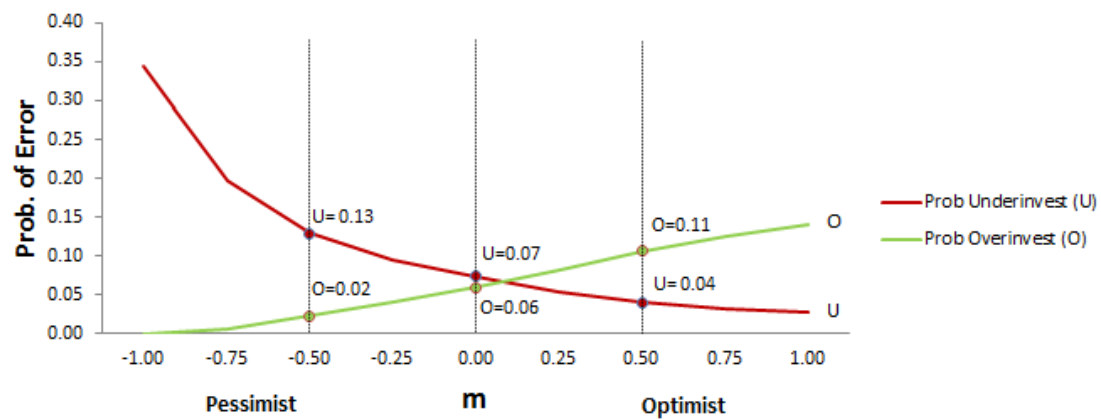
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
Paths	$\epsilon_1$	$\epsilon_2$	$\epsilon^*$	Noise	Noise*	$\Delta S$	$\Delta S^*$	$S_T$ (true)	$S_T^*$ (observed)	$B_T$ (Belief)	Full info (C) max( $S_T-K,0$ )	Perceived (Cp) max( $B_T-K,0$ )	Behavioral (C*) $B_T > K$ get $S_T-K$ ; 0	Overinvest (O) $B_T > K$ & $S_T-K < 0$	Underinvest (U) $B_T < K$ & $S_T-K > 0$	Total Error (T) Total Error
1	0.84	-0.16	0.69	0.25	0.18	0.23	0.10	125.39	110.39	109.35	15.39	0.00	0.00	0.00	15.39	15.39
2	-0.45	-0.77	-0.74	-0.13	-0.19	-0.16	-0.27	85.31	76.27	78.65	0.00	0.00	0.00	0.00	0.00	0.00
3	-0.37	-0.35	-0.49	-0.11	-0.13	-0.14	-0.21	87.26	81.39	83.25	0.00	0.00	0.00	0.00	0.00	0.00
4	0.19	-0.84	-0.19	0.06	-0.05	0.03	-0.13	103.38	87.87	89.08	0.00	0.00	0.00	0.00	0.00	0.00
5	-0.27	2.20	0.72	-0.08	0.19	-0.11	0.11	89.98	111.30	110.17	0.00	0.17	-20.02	20.02	0.00	20.02
6	0.44	0.44	0.59	0.13	0.15	0.11	0.07	111.26	107.63	106.87	1.26	0.00	0.00	0.00	1.26	1.26
7	1.68	-0.95	1.10	0.50	0.29	0.48	0.21	161.49	122.86	120.58	51.49	10.58	51.49	0.00	0.00	0.00
8	1.30	-1.10	0.69	0.39	0.18	0.36	0.10	143.95	110.44	109.40	33.95	0.00	0.00	0.00	33.95	33.95
9	-1.53	-1.58	-2.06	-0.46	-0.54	-0.48	-0.61	61.71	54.10	58.69	0.00	0.00	0.00	0.00	0.00	0.00
10	-1.63	-0.40	-1.65	-0.49	-0.43	-0.52	-0.51	59.72	60.25	64.22	0.00	0.00	0.00	0.00	0.00	0.00
11	-0.40	0.96	0.05	-0.12	0.01	-0.15	-0.07	86.38	93.66	94.29	0.00	0.00	0.00	0.00	0.00	0.00
12	0.21	0.35	0.35	0.06	0.09	0.04	0.01	103.98	101.08	100.97	0.00	0.00	0.00	0.00	0.00	0.00
13	-0.73	-0.61	-0.93	-0.22	-0.24	-0.24	-0.32	78.33	72.65	75.38	0.00	0.00	0.00	0.00	0.00	0.00
14	0.16	0.93	0.55	0.05	0.14	0.02	0.06	102.36	106.57	105.91	0.00	0.00	0.00	0.00	0.00	0.00
15	-1.67	-0.44	-1.70	-0.50	-0.44	-0.53	-0.52	59.04	59.43	63.49	0.00	0.00	0.00	0.00	0.00	0.00
16	-0.14	-0.83	-0.49	-0.04	-0.13	-0.07	-0.21	93.55	81.41	83.27	0.00	0.00	0.00	0.00	0.00	0.00
17	0.89	-0.69	0.50	0.27	0.13	0.24	0.05	127.37	105.25	104.72	17.37	0.00	0.00	0.00	17.37	17.37
18	-1.65	-0.60	-1.74	-0.49	-0.45	-0.52	-0.53	59.50	58.74	62.87	0.00	0.00	0.00	0.00	0.00	0.00
19	0.03	-0.28	-0.09	0.01	-0.02	-0.02	-0.10	98.50	90.22	91.20	0.00	0.00	0.00	0.00	0.00	0.00
20	-1.36	-0.36	-1.39	-0.41	-0.36	-0.43	-0.44	64.79	64.47	68.02	0.00	0.00	0.00	0.00	0.00	0.00
21	-0.61	-1.54	-1.22	-0.18	-0.32	-0.21	-0.40	81.16	67.28	70.56	0.00	0.00	0.00	0.00	0.00	0.00
22	1.04	0.93	1.34	0.31	0.35	0.29	0.27	133.24	130.97	127.87	23.24	17.87	23.24	0.00	0.00	0.00
23	-0.21	-2.02	-1.07	-0.06	-0.28	-0.09	-0.36	91.71	70.04	73.04	0.00	0.00	0.00	0.00	0.00	0.00
24	1.42	1.32	1.86	0.43	0.48	0.40	0.40	149.51	149.68	144.71	39.51	34.71	39.51	0.00	0.00	0.00
25	-1.33	0.38	-1.04	-0.40	-0.27	-0.43	-0.35	65.37	70.58	73.52	0.00	0.00	0.00	0.00	0.00	0.00
<b>Avg (disc) 20000 paths</b>											<b>8.88</b>	<b>4.17</b>	<b>7.27</b>	<b>0.16</b>	<b>1.45</b>	<b>1.61</b>

Figure 2. Variation of behavioral option value ( $C^*$ ) and misinvestment errors (Underinvestment U, Overinvestment O and Total error T) (Panel A), along with the probability (Panel B) and average size of each mis-investment error (Panel C) for different degrees of behavioral bias (pessimism vs. optimism) when info. precision is very high ( $\rho = 0.9$ ). Option value ( $C^*$ ) is highest for an unbiased manager ( $m = 0$ ).

Panel A. Option value and mis-investment losses.



Panel B. Probability of misinvestment errors.



Panel C. Average size of misinvestment errors.

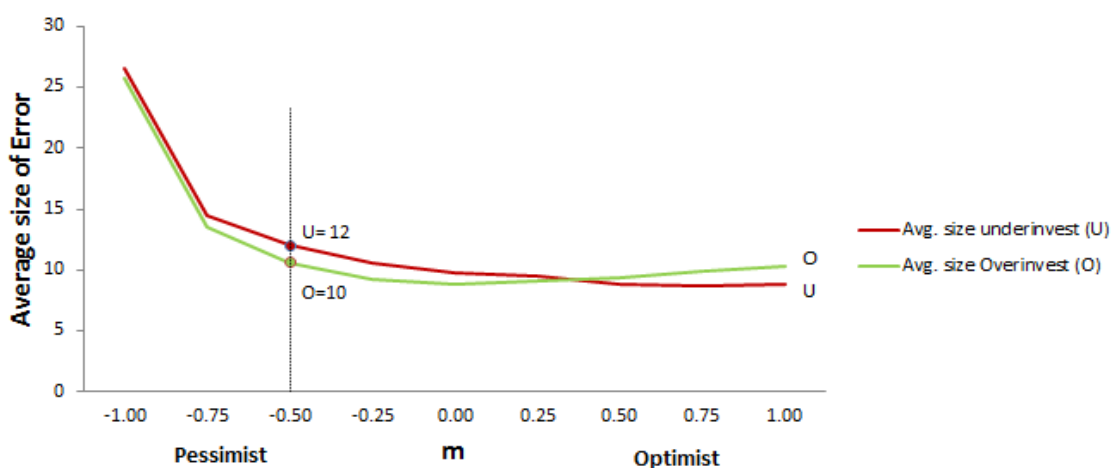
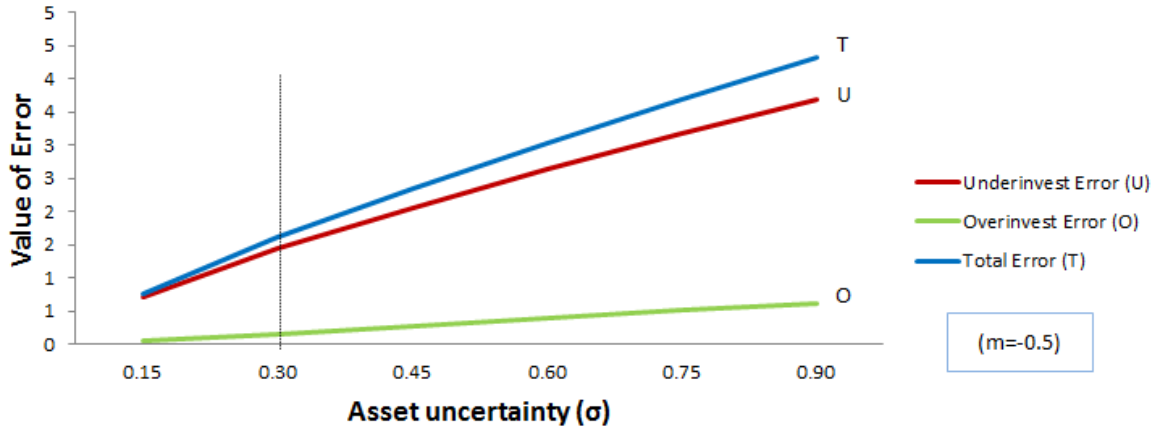


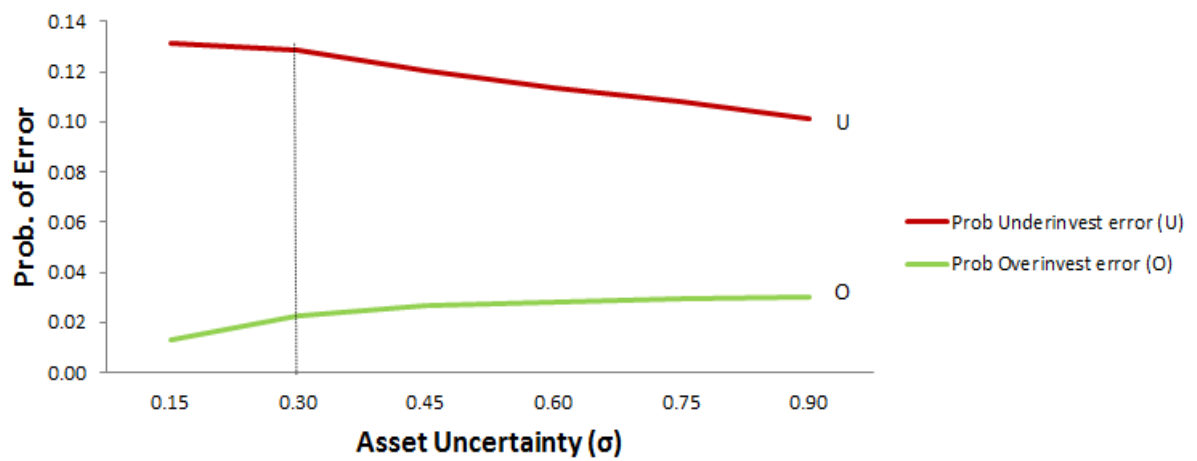


Figure 3. Variation of value of misinvestment errors (losses) (Panel A), probability of misinvestment errors (Panel B) and average size of errors (Panel C) with asset (project) uncertainty ( $\sigma$ ) for a pessimist manager ( $m = -0.5$ ) when information precision is high ( $\rho = 0.9$ ).

Panel A. Value of misinvestment errors (losses).



Panel B. Probability of misinvestment errors.



Panel C. Average size of misinvestment errors.

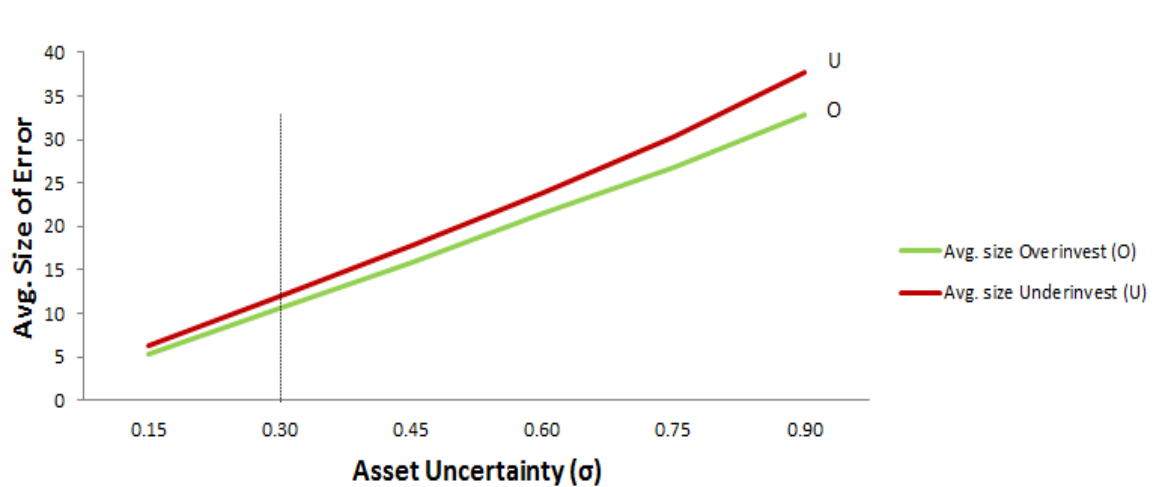


Figure 4. Behavioral option value ( $C^*$ ) variation with asset (project) uncertainty ( $\sigma$ ) at different degrees of information precision ( $\rho$ ) for an unbiased ( $m = 0$ ), pessimist ( $m = -0.5$ ) and optimist ( $m = +0.5$ ) manager. Option value becomes negative at low  $\rho$ .

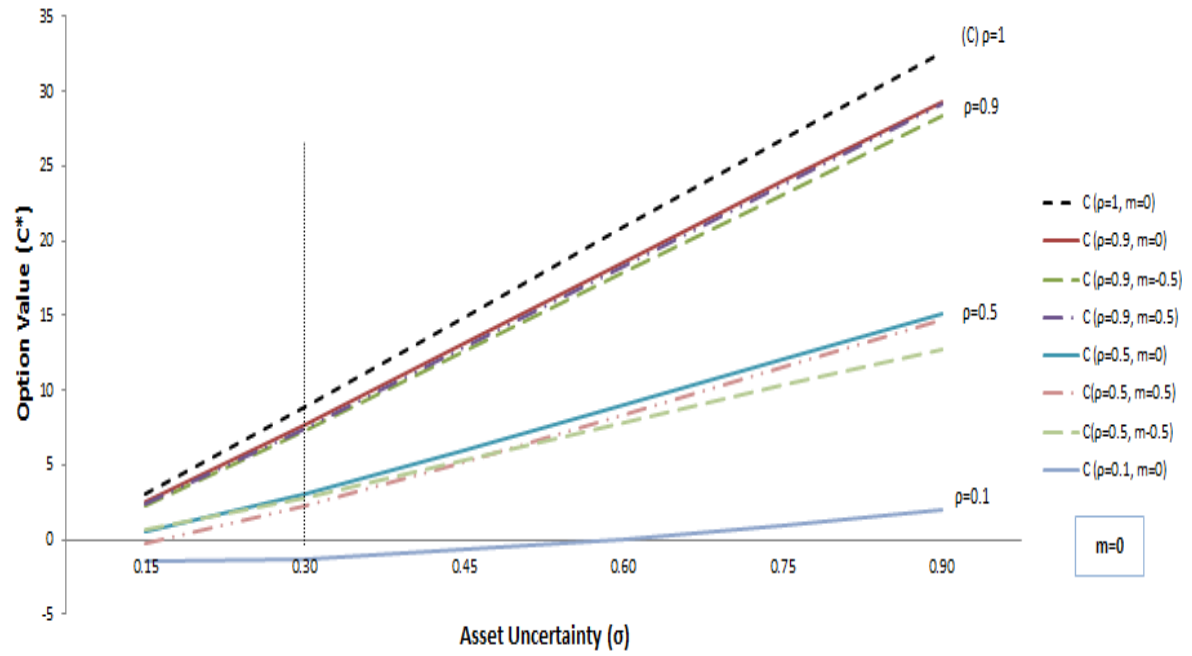
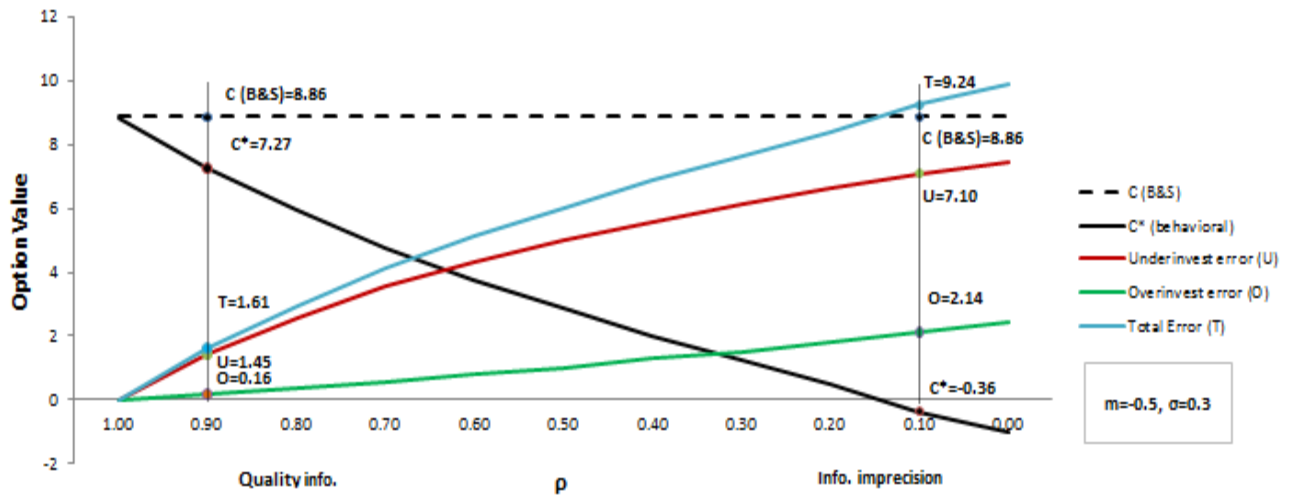


Figure 5. Variation of option value, misinvestment losses and probability of misinvestment errors with information imprecision ( $\rho$ ) for a pessimist manager ( $m = -0.5$ ).

Panel A. Behavioral option value ( $C^*$ ) with Underinvestment (U), Overinvestment (O) and Total error (T) as the degree of information imprecision rises ( $\rho$  declines). At low  $\rho$  (0.10) total mis-investment error (T) exceeds Black&Scholes theoretical option value (C) and  $C^*$  becomes negative.



Panel B. Probability of Underinvestment (U) and Overinvestment (O) error as information imprecision rises

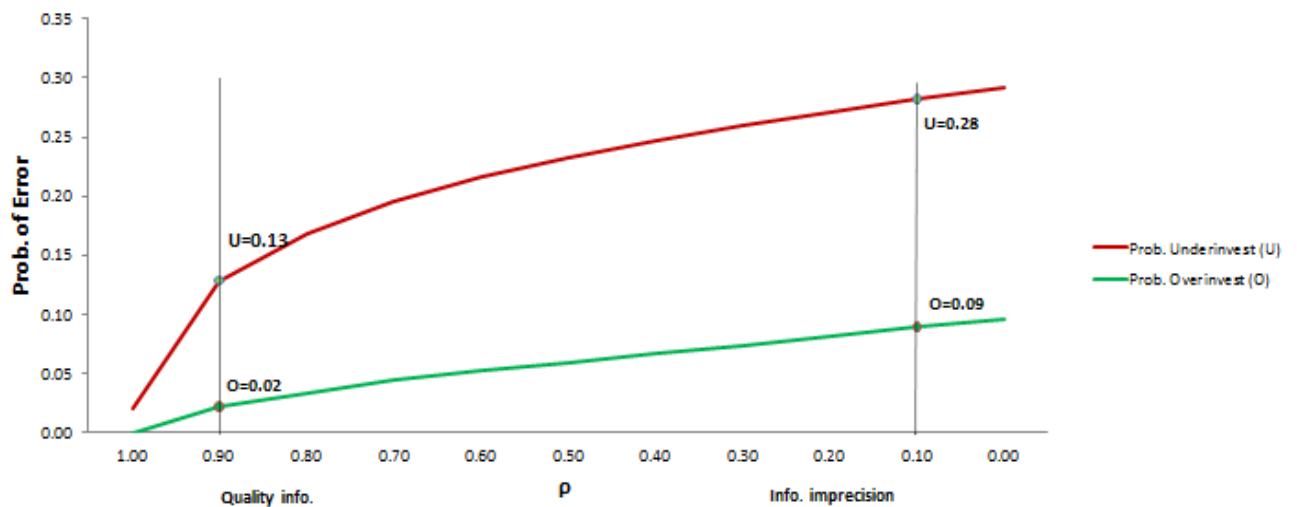


Figure 6. Anatomy of Underinvestment (U) and Overinvestment errors (O) for an overconfident manager ( $m=+0.5$ ) acting on beliefs when information precision is low ( $\rho = 0.1$ ) with a high learning rate ( $\alpha = 0.7$ ).

A	I	J	K	L	M	N	O	P	Q
Path	$S_T$ (true)	$S_T^*$ (observed)	$B_T$ (Belief)	Full info (C) $\max(S_T-K, 0)$	Perceived ( $C_p$ ) $\max(B_T-K, 0)$	Behavioral ( $C^*$ ) $B_T > K$ get $S_T-K$ else 0	Overinvest (O) $B_T > K$ & $S_T-K < 0$	Underinvest (U) $B_T < K$ & $S_T-K > 0$	Total Error (T) O+U
	1	125.39	109.25	106.47	15.39	0.00	0.00	0.00	15.39
2	85.31	85.26	89.68	0.00	0.00	0.00	0.00	0.00	0.00
3	87.26	98.27	98.79	0.00	0.00	0.00	0.00	0.00	0.00
4	103.38	85.01	89.51	0.00	0.00	0.00	0.00	0.00	0.00
5	178.50	115.94	111.16	68.50	1.16	68.50	0.00	0.00	0.00
6	58.89	64.79	75.35	0.00	0.00	0.00	0.00	0.00	0.00
7	161.49	86.16	90.31	51.49	0.00	0.00	0.00	51.49	51.49
8	143.95	80.92	86.65	33.95	0.00	0.00	0.00	33.95	33.95
9	61.71	62.88	74.02	0.00	0.00	0.00	0.00	0.00	0.00
10	59.72	92.76	94.93	0.00	0.00	0.00	0.00	0.00	0.00
11	86.38	151.98	136.38	0.00	26.38	-23.62	23.62	0.00	23.62
12	103.98	126.88	118.81	0.00	8.81	-6.02	6.02	0.00	6.02
13	78.33	89.01	92.31	0.00	0.00	0.00	0.00	0.00	0.00
14	102.36	153.41	137.39	0.00	27.39	-7.64	7.64	0.00	7.64
15	59.04	91.38	93.96	0.00	0.00	0.00	0.00	0.00	0.00
16	93.55	84.48	89.14	0.00	0.00	0.00	0.00	0.00	0.00
17	127.37	91.73	94.21	17.37	0.00	0.00	0.00	17.37	17.37
18	59.50	86.78	90.75	0.00	0.00	0.00	0.00	0.00	0.00
19	98.50	102.16	101.51	0.00	0.00	0.00	0.00	0.00	0.00
20	64.79	94.76	96.33	0.00	0.00	0.00	0.00	0.00	0.00
21	81.16	65.72	76.00	0.00	0.00	0.00	0.00	0.00	0.00
22	133.24	158.32	140.82	23.24	30.82	23.24	0.00	0.00	0.00
23	91.71	56.62	69.64	0.00	0.00	0.00	0.00	0.00	0.00
24	149.51	182.44	157.71	39.51	47.71	39.51	0.00	0.00	0.00
25	65.37	121.36	114.95	0.00	4.95	-44.63	44.63	0.00	44.63
<b>Avg (disc) 20000 paths</b>				<b>8.88</b>	<b>12.22</b>	<b>-2.39</b>	<b>7.25</b>	<b>4.01</b>	<b>11.26</b>

Figure 7. Valuation of American call option with 2 steps (A2) and 4 steps (A4) for an optimist ( $m=+0.5$ ) with high information precision ( $\rho=0.9$ ) and high learning rate ( $\alpha=0.7$ ).

Panel A. Evolution of values and beliefs

A	B	C	D	E	F	G	H	I
	S <sub>t</sub> (True)		NPV <sub>2</sub>	S* <sub>t</sub> (Observed)		B <sub>t</sub> (Beliefs)		NPV <sub>2</sub> <sup>B</sup>
Path	1	2	(S <sub>2</sub> -K)	1	2	1	2	(B <sub>2</sub> -K)
1	92.20	118.63	8.63	84.86	116.14	92.40	109.02	-0.98
2	127.12	140.87	30.87	133.53	168.94	126.47	156.20	46.20
3	69.58	85.65	-24.35	90.17	120.59	96.12	113.25	3.25
4	71.41	49.44	-60.56	79.35	47.38	88.54	59.73	-50.27
5	120.35	102.23	-7.77	152.72	153.28	139.91	149.27	39.27
6	137.35	106.98	-3.02	146.27	116.11	135.39	121.89	11.89
7	117.84	99.63	-10.37	138.70	101.42	130.09	110.02	0.02
8	134.40	125.77	15.77	169.44	150.96	151.61	151.15	41.15
9	105.06	94.36	-15.64	136.67	129.67	128.67	129.37	19.37
10	113.47	131.81	21.81	118.07	155.13	115.65	143.28	33.28

Panel B. Full information and behavioral option values at t=1

A	B	C	D	E	F	G	H	I
	Full Info. American Option				Behavioral American Option			
Path	S <sub>t</sub>	NPV <sub>t</sub>	C <sub>t+1</sub>	E(C <sub>t</sub> )	B <sub>t</sub>	NPV <sub>t</sub> <sup>B</sup>	C <sub>t+1</sub> <sup>B</sup>	E(C <sub>t</sub> <sup>B</sup> )
1	0.00	0.00	8.55	0.00	0.00	0.00	0.00	0.00
2	127.12	17.12	30.56	23.41*	126.47	16.47	45.74	26.54*
3	0.00	0.00	0.00	0.00	0.00	0.00	3.22	0.00
4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	120.35	10.35	0.00	22.13*	139.91	29.91*	38.87	29.44
6	137.35	27.35*	0.00	25.33	135.39	25.39	11.77	28.47*
7	117.84	7.84	0.00	21.66*	130.09	20.1	0.02	27.32*
8	134.40	24.40	15.62	24.78*	151.61	41.61*	40.74	31.97
9	0.00	0.00	0.00	0.00	128.67	18.7	19.18	27.01*
10	113.47	3.47	21.60	20.83*	115.65	5.7	32.95	24.2*
			Const.	-0.534			Const.	-0.780
			S <sub>t</sub>	0.188			B <sub>t</sub>	0.216

Panel C. Full information and behavioral option values, along with overinvest (O), underinvest (U) and total errors (T) at t=0 for American option with 2 steps A2).

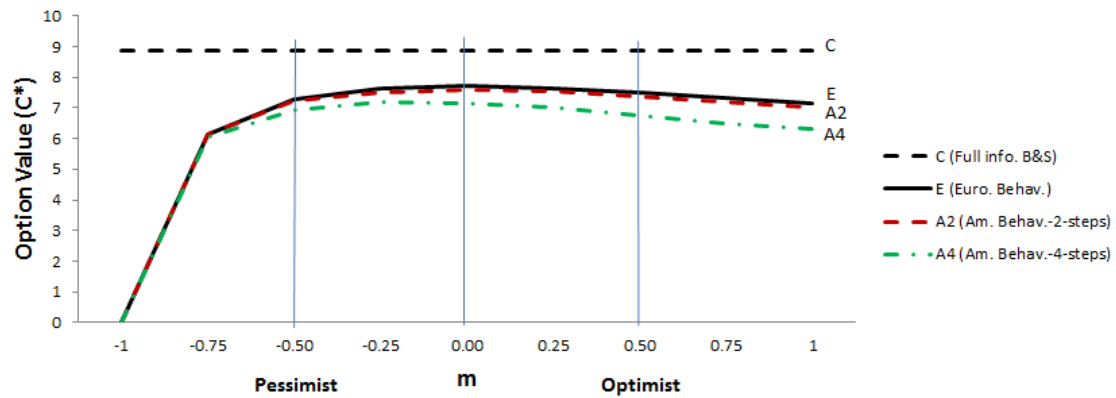
A	B	C	D	E	F	G	H	I	J	K	L	M	N	
	Full Info. NPV <sub>t</sub> at Option exer.			Behavioral Option NPV <sub>t</sub> at Belief			Overinvest Error (O)			Underinvest Error (U)			Total Error (T)	
	If NPV <sub>t</sub> > E(C <sub>t</sub> ) get NPV <sub>t</sub> ; 0			If NPV <sub>t</sub> <sup>B</sup> > E(C <sub>t</sub> <sup>B</sup> ) get NPV <sub>t</sub> <sup>B</sup> ; 0										
Path	1	2	Σ <sub>t</sub> <sup>2</sup>	1	2	Σ <sub>t</sub> <sup>2</sup>	1	2	Σ <sub>t</sub> <sup>2</sup>	1	2	Σ <sub>t</sub> <sup>2</sup>	(T)	
1	0.00	8.46	8.46	0.00	0.00	0.00	0.00	0.00	0.00	0.00	8.46	8.46	8.46	
2	0.00	30.26	30.26	0.00	30.26	30.26	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
3	0.00	0.00	0.00	0.00	-23.87	-23.87	0.00	23.87	23.87	0.00	0.00	0.00	23.87	
4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
5	0.00	0.00	0.00	10.24	0.00	10.24	-10.24	0.00	-10.24	0.00	0.00	0.00	-10.24	
6	27.08	0.00	27.08	0.00	-2.96	-2.96	0.00	2.96	2.96	27.08	0.00	27.08	30.04	
7	0.00	0.00	0.00	0.00	-10.17	-10.17	0.00	10.17	10.17	0.00	0.00	0.00	10.17	
8	0.00	15.46	15.46	24.16	0.00	24.16	-24.16	0.00	-24.16	0.00	15.46	15.46	-8.69	
9	0.00	0.00	0.00	0.00	-15.33	-15.33	0.00	15.33	15.33	0.00	0.00	0.00	15.33	
10	0.00	21.38	21.38	0.00	21.38	21.38	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
	Avg (disc) 20000 paths						7.32	-0.14	-0.25	-0.39	1.20	0.46	1.66	1.27

Panel D. Valuation of American option with 4 steps (A4): Full info and behavioral option values, along with overinvest (O), underinvest (U) and total errors (T) at t=0.

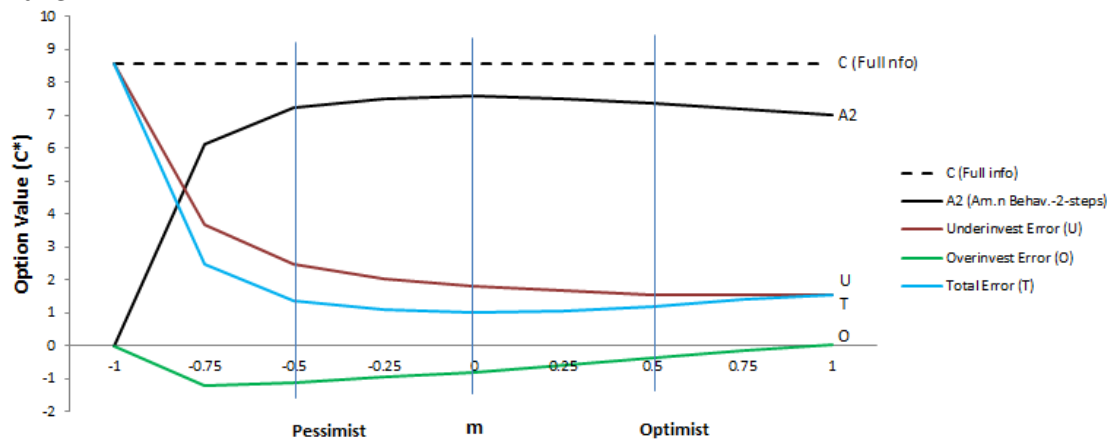
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	
	Full Info. NPVt at Option exer. <i>If <math>NPV_t &gt; E(C_t)</math> get <math>NPV_t; 0</math></i>					Behavioral Option NPVt at Belief <i>If <math>NPV_t^B &gt; E(C_t^B)</math> get <math>NPV_t; 0</math></i>					Overinvest Error (O)					Underinvest Error (U)					Total Error (T)	
Path	1	2	3	4	$\Sigma_1^4$	1	2	3	4	$\Sigma_1^4$	1	2	3	4	$\Sigma_1^4$	1	2	3	4	$\Sigma_1^4$	(T)	
1	46.12	0.00	0.00	0.00	46.12	46.12	0.00	0.00	0.00	46.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.00	48.37	0.00	48.37	0.00	12.08	0.00	0.00	12.08	0.00	-12.08	0.00	0.00	-12.08	0.00	0.00	48.37	0.00	48.37	0.00	36.29
3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	1.18	1.18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.18	1.18	1.18
5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-6.57	-6.57	0.00	0.00	0.00	6.57	6.57	0.00	0.00	0.00	0.00	0.00	0.00	6.57
6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.26	-1.26	0.00	0.00	0.00	1.26	1.26	0.00	0.00	0.00	0.00	0.00	0.00	1.26
7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
8	0.00	44.27	0.00	0.00	44.27	6.43	0.00	0.00	0.00	6.43	-6.43	0.00	0.00	0.00	-6.43	0.00	44.27	0.00	0.00	44.27	0.00	37.84
9	0.00	0.00	0.00	7.28	7.28	0.00	0.00	0.00	7.28	7.28	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	0.00	0.00	0.00	0.00	0.00	5.73	0.00	0.00	0.00	5.73	-5.73	0.00	0.00	0.00	-5.73	0.00	0.00	0.00	0.00	0.00	0.00	-5.73
						Avg (disc) 20000 paths					6.80	0.13	0.68	0.24	0.36	-1.41	0.53	0.71	0.67	0.72	2.64	1.23

Figure 8. Variation of American option value (A2) and investment errors (U, O and T) with behavioral bias (pessimism vs. optimism) with high info. precision ( $\rho = 0.9$ ).

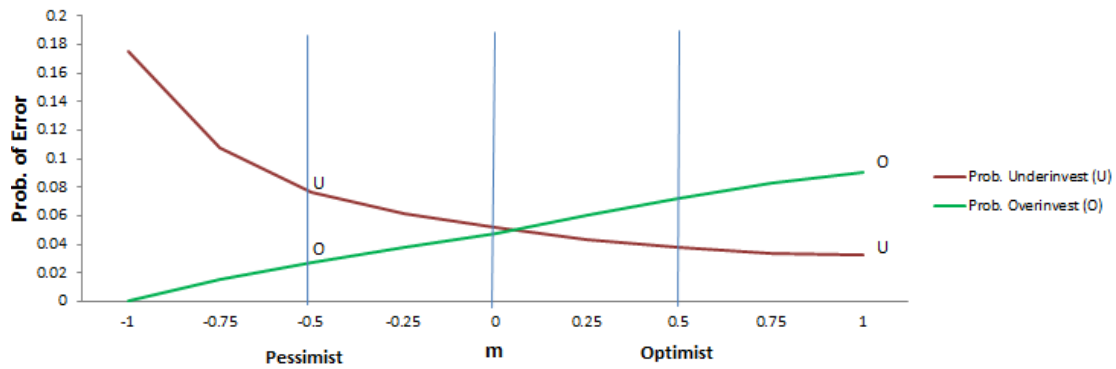
Panel A.



Panel B.



Panel C.



Panel D.

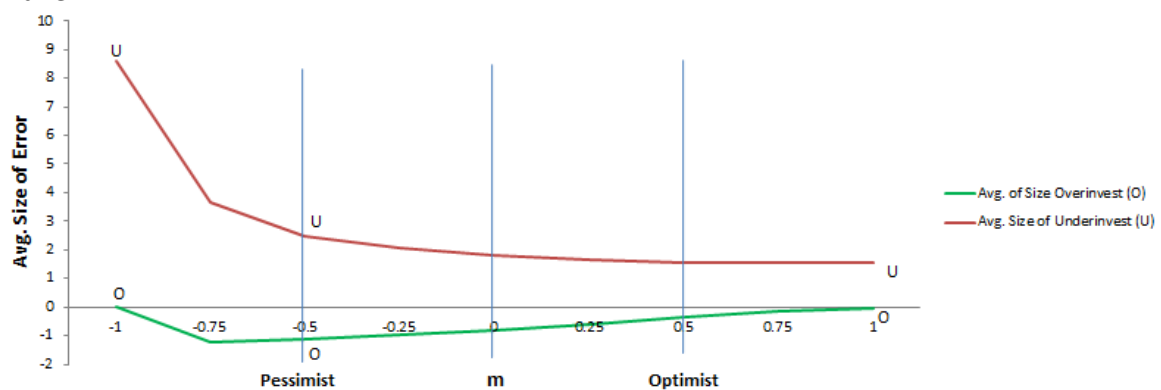
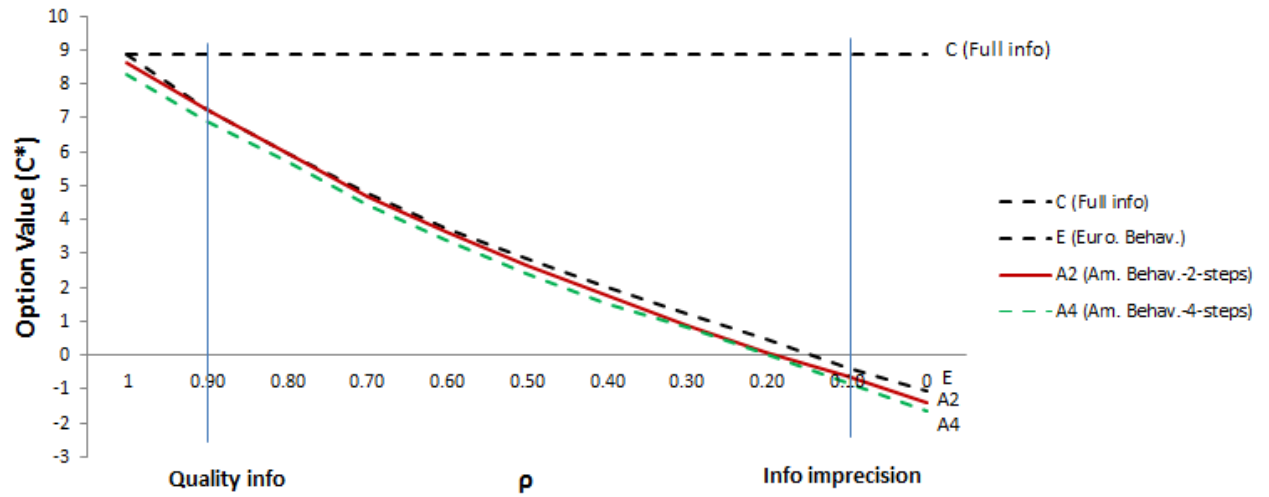


Figure 9. Variation of American option value A2), investment errors and probability of errors with information imprecision ( $\rho$ ) for a pessimist manager ( $m = -0.5$ ).

Panel A.



Panel B.

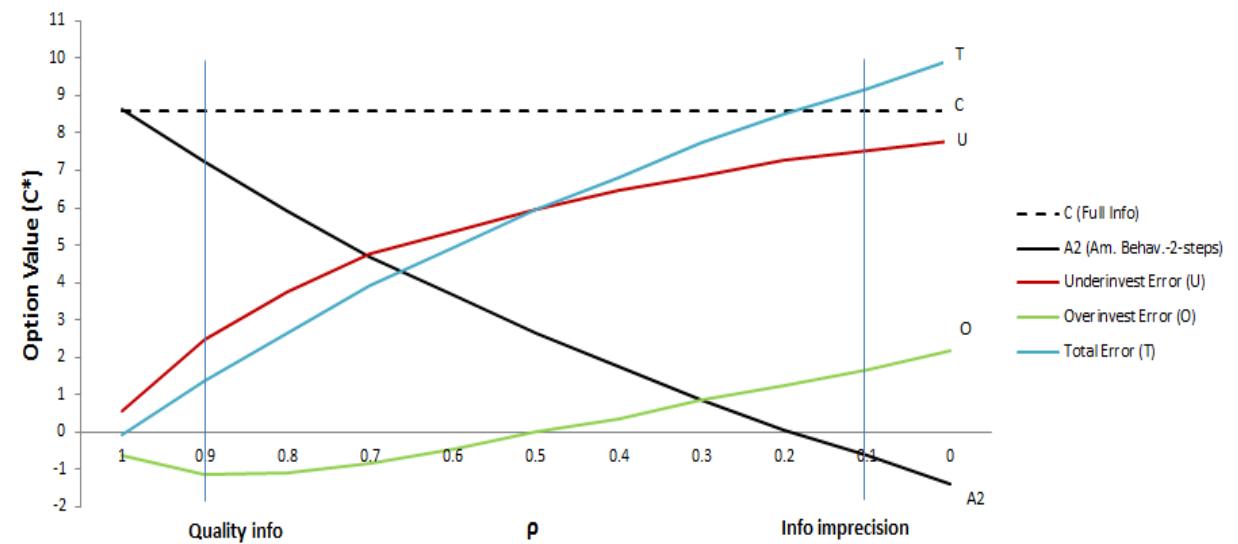
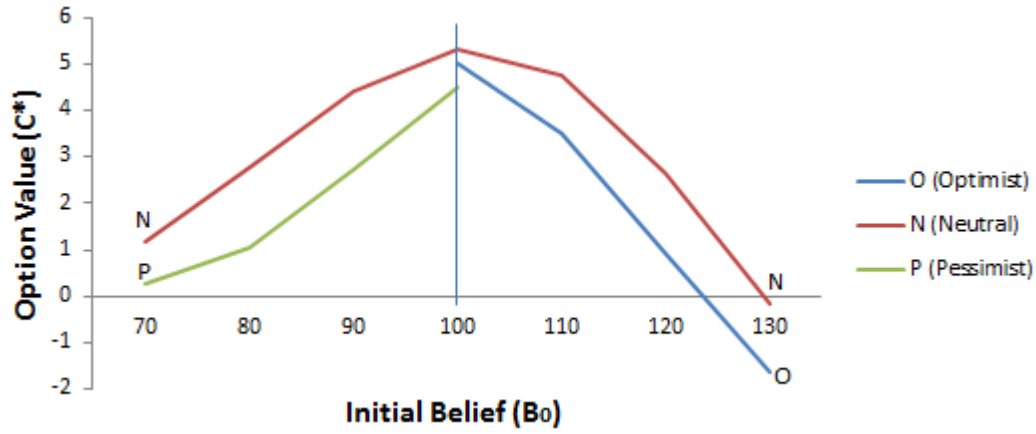




Figure 10. Sensitivity of behavioral option value ( $C^*$ ) to bias in initial beliefs ( $B_0$ ) for a pessimist ( $m = -0.5$ ), neutral ( $m = 0$ ) and optimist manager ( $m = +0.5$ ) when information precision is moderate ( $\rho = 0.7$ ).

Panel A. European option.



Panel B. American option with 4 steps (A4).

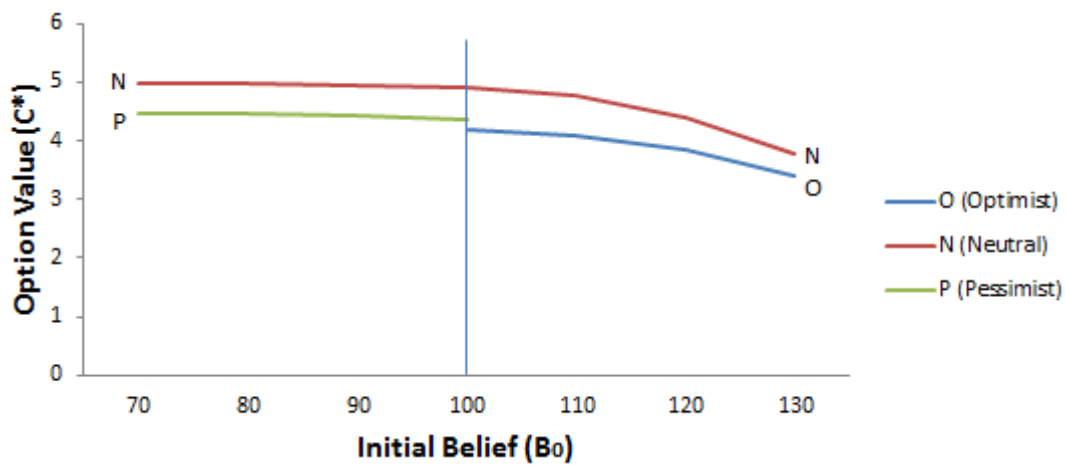
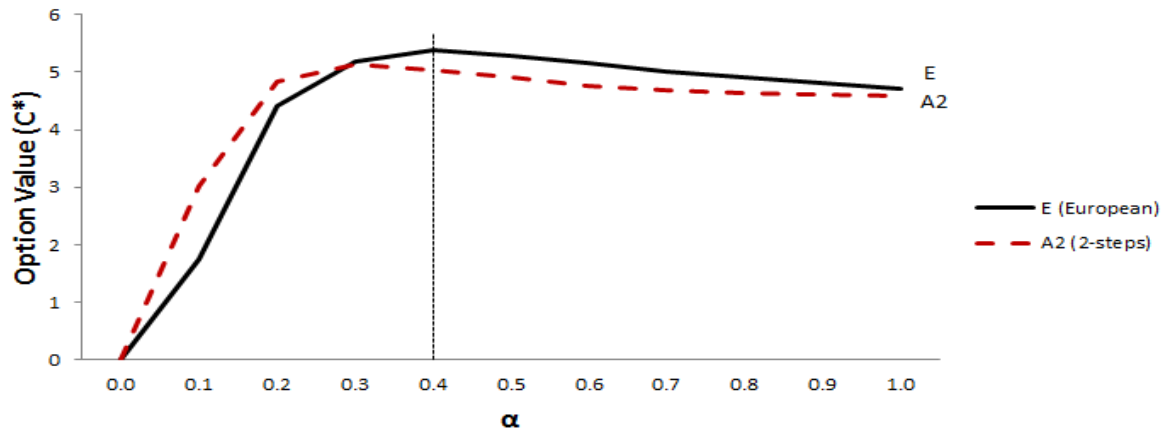
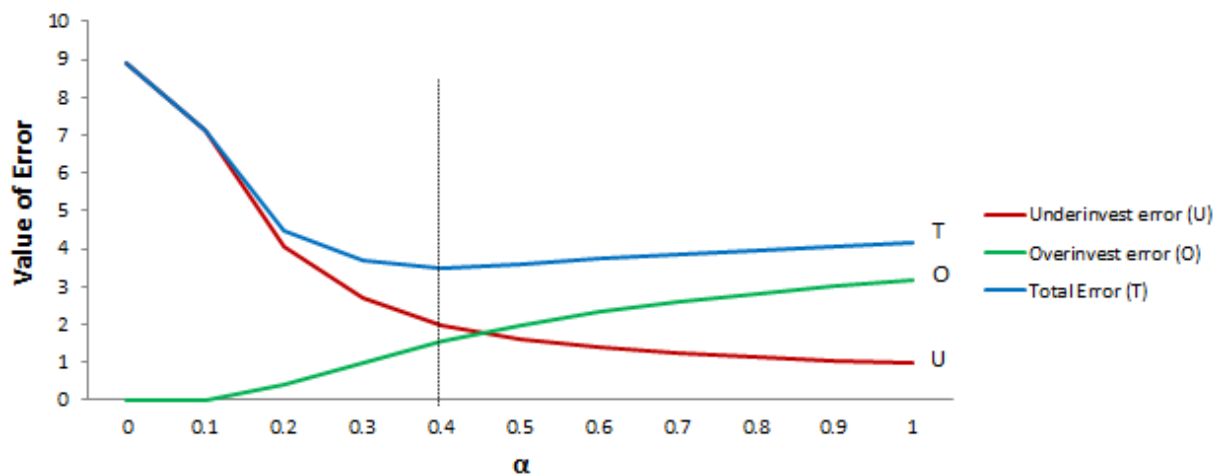


Figure 11. Variation of behavioral European call option value ( $C^*$ ) (Panel A), value of investment errors (losses) (Panel B) and probability of investment errors (Panel C) with organization learning or adjustment rate policy ( $\alpha$ ) under bounded rationality for an optimist manager ( $m = +0.5$ ) and good information quality ( $\rho = 0.7$ ). Optimal adjustment rate ( $\alpha^*$ ) around 0.4. American option with 2 steps (A2) also shown in Panel A (dotted).

Panel A. Behavioral option value ( $C^*$ ) for the European (E) and American option with 2 steps (A2).



Panel B. Value of investment errors (losses).



Panel C. Probability of mis-investment errors.

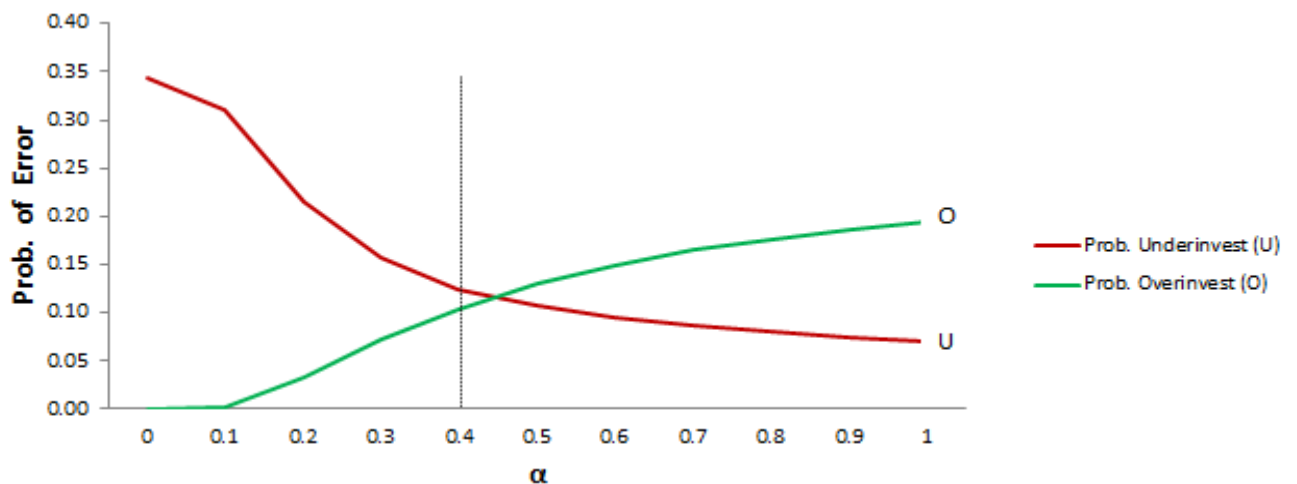


Figure 12. Sensitivity of behavioral option value to degree of myopia ( $b$ ) for an optimist ( $m = +0.5$ ) with high information precision ( $\rho=0.9$ ) and moderate learning rate ( $\alpha=0.5$ ).

