

# Project Evaluation under Uncertainty: The Hedged Monte Carlo Approach.

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Thanks: Luiz Brandão and Gláucia Fernandes



# Outline

- 1 Background
- 2 Tools
- 3 HMC Algorithm
- 4 Examples
- 5 Conclusions

# Main Question

How to evaluate projects and their optionalities under uncertainty in a consistent way with market fluctuations?



We are interested in assigning monetary values to strategic decisions. These include:

- create a new firm;
- invest in a new project;
- start a real estate development;
- finance R& D;
- temporarily suspend operations under adverse conditions.

# Project Evaluation in the Commodity Related Industries

## Common Questions

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- Complex claims
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- Exotic character
- Presence of cash flows and decision trees
- Optimal exercise times
- **Mix of historical and risk neutral measures**
- Hedgeable and unhedgeable claims.

# Historical Notes on Real Options and

## Some References

- Black-Scholes (1973) The Pricing of Options and Corporate Liabilities
- Myers (1977)
- Octavio Tourinho (IPEA-BR) (1979) Berkeley PhD thesis -The option value of reserves of natural resources.
- McDonald - Siegel (1986)
- Myers and Majd (1990)

Bibliographic References: (Myers(1977); Brennan and Schwartz(1985); McDonald and Siegel(1986); Dixit(1989); Trigeorgis and Mason(1987); Pindyck(1991); Paddock et al.(1988)Paddock, Siegel, and Smith; Tourinho(1979); Ingersoll and Ross(1992))

- 1 Lenos Trigeorgis and the *Real Options* series

# Historical Notes on Real Options in Brazil

- 1 Lenos Trigeorgis and the *Real Options* series
- 2 Group at PUC: (to cite a few)
  - Luiz Eduardo Teixeira Brandão
  - Marco Antonio Guimarães Dias
  - José Paulo Teixeira



Figure: 12th Real Option Meeting

## Concept of Spanning Asset

A **traded** asset that is *highly* correlated with the project's value  $V$ .

Thus...

$$P(t, V_t; T) = \sup_{t \leq \tau \leq T} \mathbb{E}_t \left[ e^{r(t-\tau)} (V_\tau - I)^+ \right]$$

Example of Model for  $V$ :

$$dV = (r - \delta)Vdt + \sigma VdW$$

Then  $P$  satisfies a Black-Scholes Model with Free Boundary conditions.

# Comparison between Financial & Real Options

## An Example: Investment in a New Project

Financial Option	Real Option
Underlying Price	Project's Present Value
Variance of the Stock	Variance of the Return Value
Exercise Price	Development Cost
Expiration Date	Time Limit for the Investment
Risk Free Rate	Risk Free Rate for the Investment
Dividend Rate	Risk Adjusted Return Rate of the Project



# Usual Assumptions in using Real Options

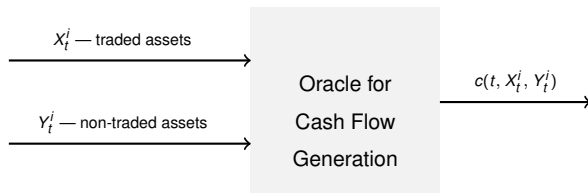
## Critique

- infinite time horizon,
- perfectly correlated spanning asset, complete market, perfect hedging...
- absence of competition.

## Critique

See (Hubalek and Schachermayer(2001)): The limitations of no-arbitrage arguments for real options

# Complex structured real options



# Motivation/Preliminaries

## Requirements and Specs

- Cash flows and project values are **highly** dependent on commodity prices.

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## Goal

Provide a methodology to evaluate *real option* decisions that would be simple (yet not too simple) to fulfill as much as possible the above requirements.

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E. Brigatti, F. Macias, M. Souza, and J. Zubelli.

A hedged Monte Carlo approach to real option pricing.

In *Commodities, Energy and Environmental Finance*, Vol. 74 of *Fields Institute Communications*. Springer, 2015.

- *Historical measure*: Simulations usually presented in historical measure. Scenarios provided by management and are loaded with views from “specialists.”
- *Managerial views*: It is crucial to incorporate managerial views in the cash flows, as well as automated decisions.
- *Market incompleteness*: The hedging is performed in incomplete financial markets.
- *Unhedgeable risks*: Non financial risks usually present.
- *Multiple assets*: Investment decisions may depend on the relative value of several traded underlyings.

- The Classical Method (eg. MAD method)
- Monte Carlo Based Approaches
- Datar-Mathews (DM) Method
- Jaimungal-Lawryshyn (JL)

# The Risk Minimization Approach

## Main Idea

Risk  $\sim$  variance of wealth balance.

$$R_t = \left\langle \left( e^{-r\Delta t} [\mathcal{V}_{t+\Delta t}(X_{t+\Delta t}) - \phi_t(X_t) \cdot (X_{t+\Delta t})] - [\mathcal{V}_t(X_t) - \phi_t(X_t) \cdot X_t] \right)^2 \right\rangle$$

Here:

- 1  $\mathcal{V}_t$  = price of the option or derivative
- 2  $\phi_t$  = hedging portfolio vector
- 3  $X_t$  = price of the traded assets

See: (Potters et al.(2001)Potters, Bouchaud, and Sestovic)

# Hedging in incomplete markets and discrete time

## Basic framework

- Assume to be in a filtered probability space  $(\Omega, \mathcal{F}_T, \mathbb{P})$  and write  $L^2(\mathbb{P}) = L^2(\Omega, \mathcal{F}_T, \mathbb{P})$ .
- Prices given by a  $d$ -dimensional stochastic process  $X$
- $\xi^N$  denotes the investment (short or long) in the numéraire asset
- $\xi$  denotes the position on  $d$  risky assets.
- $X$  and  $V$  denote *discounted prices* with respect to a risk-free process.



# Hedging in incomplete markets and discrete time

Basic vocabulary I. (Following (Föllmer and Schied(2004)))

## Definition

*Trading strategy*: pair of stochastic processes  $(\xi^N, \xi)$ , w/  $\xi_t^N$  adapted process and  $\xi$  is a  $d$ -dimensional predictable process.

The discounted value of the portfolio is

$$V_t := \xi_t^N + \xi_t \cdot X_t$$

Gain process:

$$G_t := \sum_{s=1}^t \xi_s \cdot (X_s - X_{s-1}).$$

Cost process:

$$C_t := V_t - G_t.$$

# Hedging in incomplete markets and discrete time

## Basic vocabulary II

Let  $H$  denote a random claim, and assume that

- 1  $H \in L^2(\mathbb{P})$ ;
- 2  $X_t \in L^2(\Omega, \mathcal{F}_t, \mathbb{P}; \mathbb{R}^d)$ , for all  $t$ .

### Definition

An *admissible  $L^2$ -strategy* for  $H$  is a trading strategy s.t. it is replicating, i.e.,

$$V_T = H \quad \mathbb{P} \text{ a.s.},$$

and s.t. the value process and the gain process are square-integrable, i.e.,

$$V_t, G_t \in L^2(\mathbb{P}), \forall t \in [0, T].$$

### Definition

Let  $(\xi^N, \xi)$  be an  $L^2$ -admissible strategy.

*Local risk process*

$$R_t^{\text{loc}}(\xi^N, \xi) := \mathbb{E}[(C_{t+1} - C_t)^2 | \mathcal{F}_t].$$

# Hedging in incomplete markets and discrete time

## Basic vocabulary III

### Definition

Let  $(\xi^N, \xi)$  be an  $L^2$ -admissible strategy.

*Local risk process*

$$R_t^{\text{loc}}(\xi^N, \xi) := \mathbb{E}[(C_{t+1} - C_t)^2 | \mathcal{F}_t].$$

Let  $(\hat{\xi}^N, \hat{\xi})$  be an  $L^2$ -admissible strategy with value process  $\hat{V}_t$ . This strategy *locally risk-minimizing strategy* if, for each  $t$ , we have that

$$R_t^{\text{loc}}(\hat{\xi}^N, \hat{\xi}) \leq R_t^{\text{loc}}(\xi^N, \xi), \quad \mathbb{P}\text{-a.s.}$$

for every  $L^2$ -admissible strategy whose value process  $V_t$  satisfies  $V_{t+1} = \hat{V}_{t+1}$ .

# Hedging in incomplete markets and discrete time

## Basic vocabulary IV

### Definition

A trading strategy is a **mean self-financing strategy**, if corresp. cost process is a martingale, i.e.:

$$\mathbb{E}[C_{t+1} - C_t | \mathcal{F}_t] = 0.$$

### Definition

Two adapted processes  $U$  and  $V$  are strongly orthogonal if

$$\text{cov}(U_{t+1} - U_t, V_{t+1} - V_t | \mathcal{F}_t) = 0,$$

where  $\text{cov}$  denotes the conditional covariance, i.e.,

$$\text{cov}(A, B | \mathcal{F}_t) = \mathbb{E}[AB | \mathcal{F}_t] - \mathbb{E}[A | \mathcal{F}_t] \mathbb{E}[B | \mathcal{F}_t].$$



# Hedging in incomplete markets and discrete time

The tool of the trade

## Theorem (Follmer-Schweizer)

- 1 An  $L^2$ -adm. strategy is loc. risk minimizing **iff** it is mean self-financing, and its cost process is strongly orthogonal to  $X$ .
- 2  $\exists$  a loc. risk minimizing strategy **iff**  $H$  admits the so-called Follmer-Schweizer decomposition:

$$H = c + \sum_{t=1}^T \xi_t \cdot (X_t - X_{t-1}) + L_T, \quad \mathbb{P}\text{-a.s.},$$

where  $c$  is a constant,  $\xi$  is a  $d$ -dimensional predictable process, such that  $\xi_t \cdot (X_t - X_{t-1}) \in L^2(\mathbb{P})$  for each  $t$ , and  $L$  is a square-integrable martingale that is strongly orthogonal to  $X$ , and satisfies  $L_0 = 0$ .

# Hedging in incomplete markets and discrete time

## The tool of the trade (part 2)

The locally risk-minimizing strategy  $(\hat{\xi}^N, \hat{\xi})$  is given by :

$$\begin{aligned}\hat{\xi} &= \xi \\ \hat{\xi}_t^N &= c + \sum_{s=1}^t \xi_s \cdot (X_s - X_{s-1}) + L_t - \xi_t \cdot X_t.\end{aligned}$$

Notice that the associated cost process is  $C_t = c + L_t$ .

## Algorithm

- 1 Set  $\hat{V}_T := H$ ;
- 2 For  $t = T - 1$  down to  $t = 0$  do
  - 1 Set

$$(\hat{V}_t, \hat{\xi}_{t+1}) := \operatorname{argmin}_{(V_t, \xi_{t+1})} \mathbb{E} \left[ \left( \hat{V}_{t+1} - (V_t + \xi_{t+1} \cdot (X_{t+1} - X_t)) \right)^2 \mid \mathcal{F}_t \right];$$

- 3 Set  $\hat{C}_t := \hat{V}_t - \sum_{s=1}^t \hat{\xi}_s \cdot (X_s - X_{s-1})$ ,  $t = 0, \dots, T$ ;
- 4 Set  $\hat{c} := \hat{C}_0$ ;
- 5 Set  $\hat{L}_t := \hat{C}_t - \hat{c}$ ,  $t = 0, \dots, T$ ;
- 6 Set  $\hat{\xi}_t^N := \hat{c} + \sum_{s=1}^t \hat{\xi}_s \cdot (X_s - X_{s-1}) + \hat{L}_t - \hat{\xi}_t \cdot X_t$ ,  $t = 0, \dots, T$ .



- If  $\mathbb{P}$  is a risk-neutral measure, then  $X_t$  is a square-integrable martingale.  
In this case, the Galtchouk-Kunita-Watanabe decomposition yields

$$\mathbb{E}[H|\mathcal{F}_t] = \hat{V}_0 + \sum_{s=1}^t \hat{\xi}_s \cdot (X_s - X_{s-1}) + L_t$$

- Hence

$$\mathbb{E}[H|\mathcal{F}_t] = \hat{V}_t.$$

# Consistency price interpretation II

## Definition

Let  $\mathcal{P}$  denote the set of martingale measures that are equivalent to  $\mathbb{P}$ . We say that  $\hat{\mathbb{P}} \in \mathcal{P}$  is a minimal martingale measure if

$$\mathbb{E} \left[ \left( \frac{d\hat{\mathbb{P}}}{d\mathbb{P}} \right)^2 \right] < \infty,$$

and if every square-integrable martingale under  $\mathbb{P}$ , that is strongly orthogonal to  $X$  is also a martingale under  $\hat{\mathbb{P}}$ .

## Theorem

*If there exists a minimal martingale measure  $\hat{\mathbb{P}}$ , and denoting by  $\hat{V}$  the value process of the local risk minimizing strategy, then we have that*

$$\hat{V}_t = \hat{\mathbb{E}} [H | \mathcal{F}_t].$$

# Question

How to implement this in practical situations?

- 1 Start from simulation of the scenarios in the historical measure.
- 2 Produce the cash-flows associated to the different scenarios.
- 3 Combine the Föllmer - Schweizer construction with the Longstaff-Schwartz methodology for option evaluation.
- 4 Equivalently apply the Bouchaud-Potters-Sestovic hedged Monte Carlo method.

## Idea

Decompose the conditional expectation w.r.t.  $\mathcal{F}_t$  in a basis generated by functions of  $X_t$ .

## Algorithm

- 1 Initialize the project value  $\mathcal{V}_T(X_T^i)$  for the scenarios  $i = 1, \dots, N$  for  $t = T_0 \dots T$ .
- 2 Initialize for  $t = T$  the payoff  $\widehat{V}_T(X_T^i) = (\mathcal{V}_T(X_T^i) - K)^+$  for the scenarios  $i = 1, \dots, N$ .
- 3 For  $t = T - 1, \dots, T_0$  do

- 1 Define the functions:

$$V_t(x) := \sum_{a=1}^b \gamma_t^a K_a(x) \text{ and } \xi_{t+1}(x) := \sum_{a=1}^b \psi_{t+1}^a H_a(x)$$

- 2 Solve the quadratic minimization problem for  $\gamma_t^a, \psi_{t+1}^a$ :

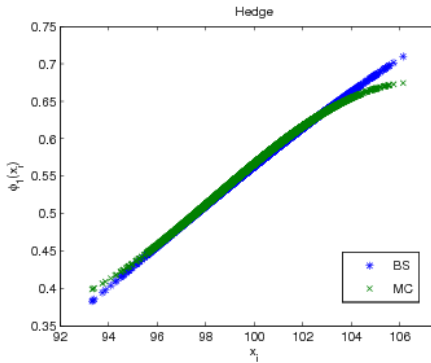
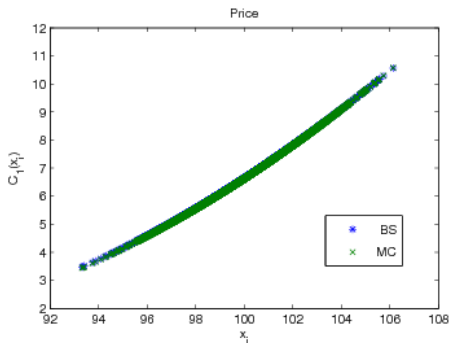
$$\operatorname{argmin}_{\{\gamma_t^a, \psi_{t+1}^a\}_{a=1}^b} \sum_{i=1}^N \left[ \rho^{-1} \widehat{V}_{t+1}(X_{t+1}^i) - \sum_{a=1}^b \gamma_t^a K_a(X_t^i) - \sum_{a=1}^b \psi_{t+1}^a H_a(X_t^i) \cdot (\rho^{-1} X_{t+1}^i - X_t^i) \right]^2$$

- 3 Define  $\widehat{V}_t(X_t^i) := \max\{(\mathcal{V}_t^i - K)^+, \widehat{V}_t(X_t^i)\}$ .

- 4 Output: The values of  $\widehat{V}_{T_0}(x)$  for  $x \in \{X_0^i\}_{i=1}^N$  and the points in the exercise region.

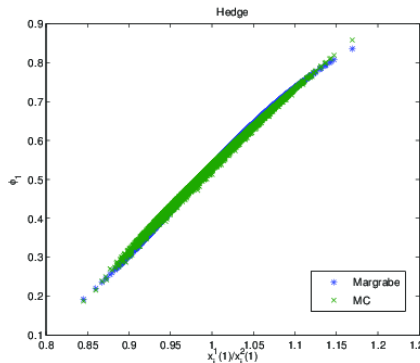
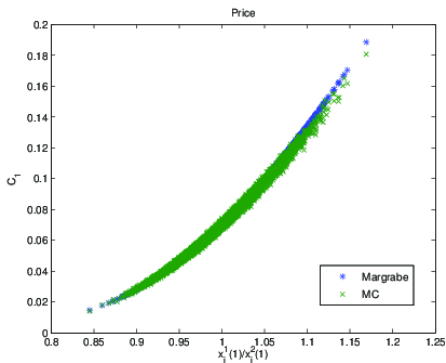
# First textbook example

- European option expiring in 3 months with strike  $K = 100$ , current asset price varying around the at-the-money value  $X(0) = 100$ , volatility  $\sigma = 0.3$ , and interest rate  $r = 0.05$ .
- The number of basis elements (monomials  $1$ ,  $x$  and  $x^2$ ) was  $b = 3$  and a total of  $N = 5000$  simulations in an arbitrary (fixed) probability measure.



# Second textbook example

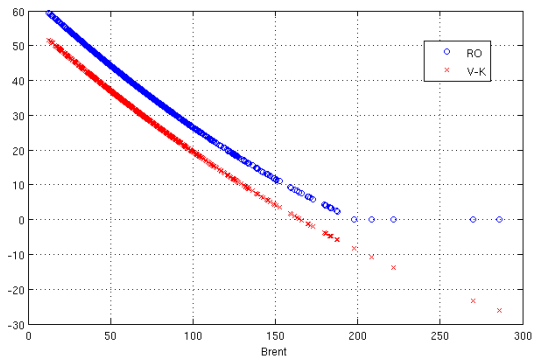
- 65 days exchange option with payoff  $(X_{1,T_F} - X_{2,T_F})^+$ .
- $X_1$  and  $X_2$  satisfy geometrical Brownian motion dynamics with  $\sigma_1 = 0.3$ ,  $\sigma_2 = 0.2$ , and  $r = 0.05$ .
- Analytical results are obtained using the Margrabe's formula:  $X_{1,0}N(d_1) - X_{2,0}N(d_2)$ , where  $N$  denotes the cumulative distribution function for a normal distribution and  $d_{1,2} = (\ln[X_{1,0}/X_{2,0}] \pm \sigma^2 T_F/2) / \sigma \sqrt{T_F}$ , with  $\sigma = \sqrt{0.3^2 + 0.2^2}$ .



# First practical example

- An energy company considers the optionality of starting a new project that would last for 11 years.
- The project value  $V_t$  is dependent on 12 different underlyings.
- The option is exercisable every year during the first 5 years.
- The company also has a trading desk that could be used for financial investment in some or all of such different assets.
- The optionality was evaluated using several different sets of hedging assets.
- Results obtained with one hedging variable (in this example the Brent price) and considering 2000 paths along 11 years with a (continuously compounded annualised) interest rate  $r = 0.08$ .

# First practical example

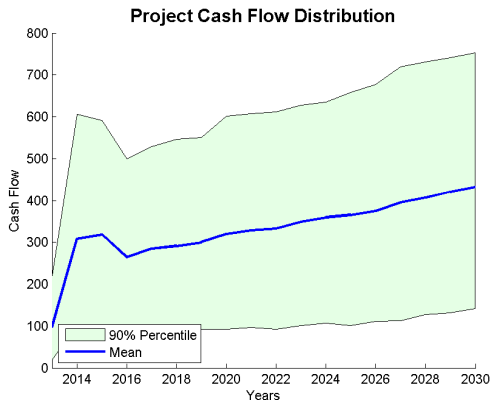


**Figure:** The investment (strike) is  $K = 10.89$  and the risk free interest rate  $r = 0.08$ .



# Second practical example

- Consider a project that would run for 15 years;
- An investment of 1500 monetary units and a yearly free interest rate of 8%.
- The cash flows for this period are the results of an oracle that depends on a number of traded and non-tradable variables and in turn are produced by means of running different scenarios.

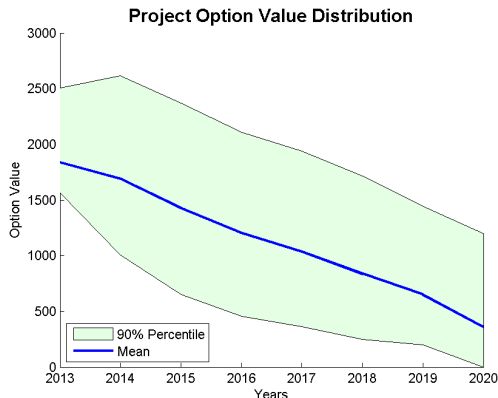


**Figure:** A description of the cash flow under the different scenarios. The lower line corresponds to the 5% quantile and the upper line to the 95%.



# Second practical example

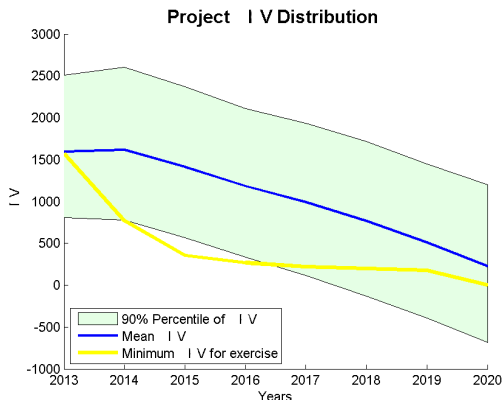
## Pricing



**Figure:** Value of the project optionality. The lower line corresponds to the 5% quantile and top one to the 95%. The marked region indicates the 90% frequency region.

# Second practical example

## Intrinsic value statistics



**Figure:** The lower line corresponds to the 5% quantile and top one to the 95%. The marked region indicates the 90% frequency region.

# Third practical example

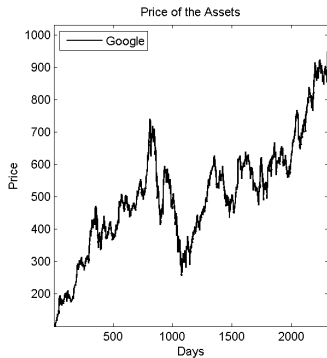
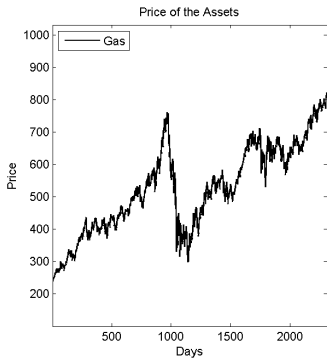
- Fictitious project where the cash flows would come from a (fairly) simple mathematical function. It concerns an artificial potential investment on a gas propelled vehicle that could be used by an information technology company to gather geographical data and to use in their web-based advertisements.
- For simplicity we take the cash flow highly correlated to Google stock through the equation

$$c_t(X, \epsilon) = H(aX_{1,t} - bX_{2,t} - l + \epsilon_t) ,$$

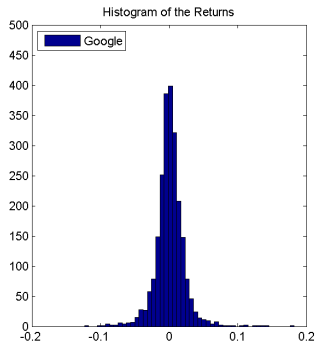
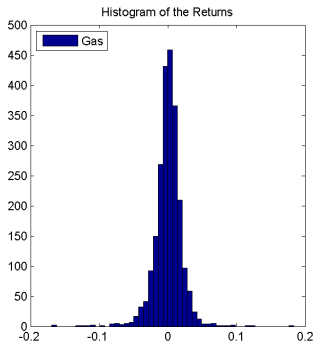
where  $X_1$  is the price of a Google stock,  $X_2$  is Henry Hub (HH) gas index,  $l$  is a fixed running cost,  $\epsilon_t$  is a nonhedgeable noise. The function  $H$  in our example is defined as

$$H(x) = \begin{cases} 0, & x \leq 0, \\ x, & x \in (0, 1), \\ 1, & x \geq 1. \end{cases}$$

- The rationale behind  $H$  is to simulate the saturation given by very large values of the stock and to clip the values below zero.



**Figure:** Time series for the assets between August 19th, 2004 and November 24th, 2013.



**Figure:** Histogram of the log returns for the assets between August 19th, 2004 and November 24th, 2013.

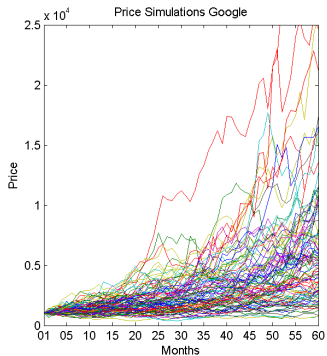
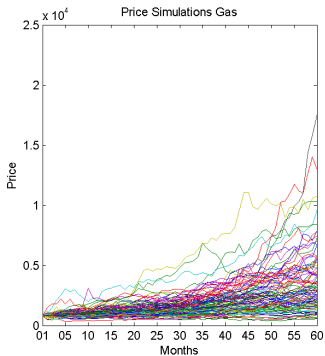
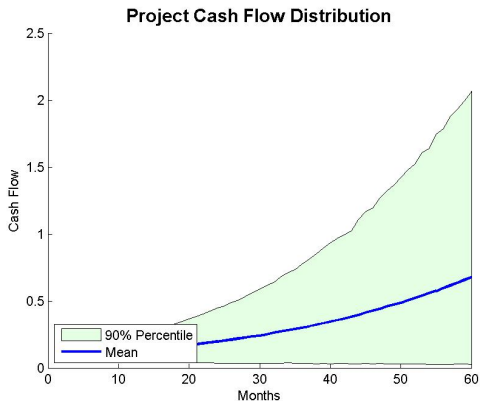
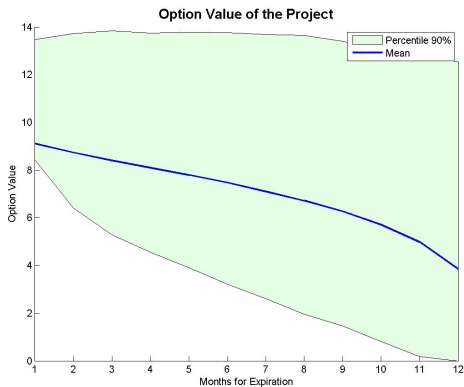


Figure: Asset simulations.

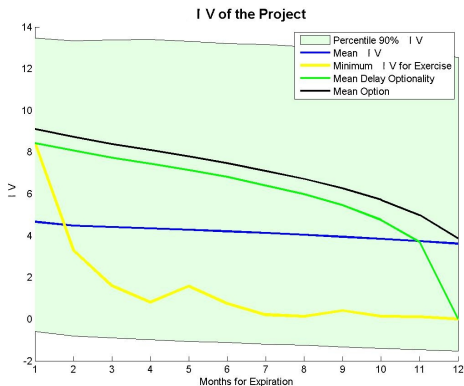


**Figure:** Cash flow simulations for the fictitious oracle described by Equation (37). Using the parameters value  $a = 1.2895 \times 10^{-4}$ ,  $b = -5.3191 \times 10^{-5}$ ,  $l = 0.05$ ,  $\varepsilon_t \sim \mathcal{N}(0, 0.005)$





**Figure:** A description of the option value statistics under the different scenarios.



**Figure:** A description of the project Intrinsic Value statistics under the different scenarios and the minimum value for exercise.

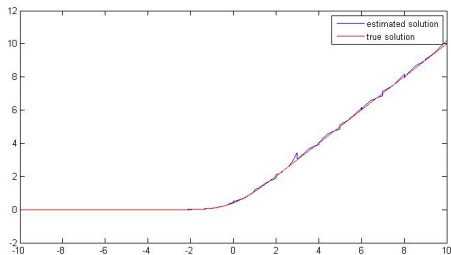
- We implemented a Real Option pricing model based on a Hedged Monte Carlo approach which :
  - ① incorporates managerial views in the cash flows
  - ② allows to by-pass the problem of using risk neutral simulations.
  - ③ can take into account competition games
- **We implemented the computation of the deferment option and of the expansion option.**
- The methodology is model free since it only depends on the simulations of scenarios for the different variables.
- For More Details See: (Brigatti et al.(2015)Brigatti, Macias, Souza, and Zubelli)

# Further Development and Ongoing Work

- Use risk measures instead of variance. (PhD thesis of F. Macias)
- Calibration and scenario generation.
- Search of optimal basis functions in high dimensional cases.
- Optimizing the computational process.
- **Ongoing work w/ E. Gobet and G. Liu (Polytechnique Paris)**
- Theoretical aspects of the methodology (such as convergence, complexity and robustness).

# Hot out of the press...

Approximation 20 subintervals of a call via Monte Carlo



**THANK YOU FOR YOUR ATTENTION!!!**

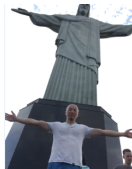


Figure: Collaborators

Thank you!

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